Rebuttal to: Has dark energy really been discovered in the Lab?

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We argue that a recent discussion of Jetzer and Straumann [Phys. Lett. B **606**, 77 (2005)] relating the measured noise spectrum in Josephson junctions to van der Waals forces is incorrect. The measured noise spectrum in Josephson junctions is a consequence of the fluctuation dissipation theorem and the Josephson effect and has nothing to do with van der Waals forces. Consequently, the argument of Jetzer and Straumann does not shed any light on whether dark energy can or cannot be measured using superconducting Josephson devices.

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I. INTRODUCTION

Recently we hypothesized that if vacuum fluctuations underly dark energy then this effect could be detected experimentally using resistively shunted Josephson junctions [1]. Our suggestion was based on an experiment by Koch et al. [2], who have shown that superconducting Josephson devices have a noise spectrum consistent with theoretical predictions [3] based on a generalized treatment of quantum fluctuations by Callen and Welton [4]. Subsequently, our paper was criticized by Jetzer and Straumann [5], who claimed there is no basis for our hypothesis.

In this note we argue that the logic behind the Jetzer and Straumann [5] criticism is misleading.

II. THE DATA AND THE THEORY

Koch et al. [3] derived the power spectrum $S(\omega)$ (units of A^2/Hz) describing the measured current noise in a resistively shunted Josephson junction in the form

$$S(\omega) = \frac{4}{R}\left[\frac{\hbar\omega}{2} + \frac{\hbar\omega}{\exp(\hbar\omega/kT) - 1}\right], \qquad (2.1)$$

where R (ohms) is the shunt resistor and T is the absolute temperature. The experimental work of Koch et al. [2] convincingly demonstrated that Equation (2.1) fits the experimental data $S(\omega)$ as a function of $\omega = 2\pi\nu$ between $\nu = 0$ and $\nu = 6 \times 10^{11}$ Hz at 1.6 and 4.2 K.

From a formal point of view, the expression in brackets of Equation (2.1) is the mean energy

$$\bar{U}(\nu, T) = \frac{1}{2}h\nu + \frac{h\nu}{\exp(h\nu/kT) - 1}, \qquad (2.2)$$

of an oscillator with frequency ν at temperature T. For low temperatures the spectrum $S(\omega)$ is dominated by the linear term in ω, which can be attributed to the effects of vacuum (zero-point) fluctuations [6]. As the temperature is increased the second term, which is identical to the ordinary Bose-Einstein statistics, plays an ever larger role in $S(\omega)$.

III. THE HYPOTHESIS AND THE CRITICISM

If we take the customary expression for the energy per unit volume at a frequency ν and temperature T

$$\rho(\nu, T) = \frac{8\pi\nu^2}{c^3}\bar{U}(\nu, T) \qquad (3.3)$$

then

$$\rho(\nu, T) = \frac{8\pi\nu^2}{c^3}\left[\frac{1}{2}h\nu + \frac{h\nu}{\exp(h\nu/kT) - 1}\right] \qquad (3.4)$$

In Equation (3.4) the first term

$$\rho_{vac}(\nu) = \frac{4\pi h\nu^3}{c^3} \qquad (3.5)$$

is due to the zeropoint fluctuations, while the second term

$$\rho_{rad}(\nu, T) = \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp\left(h\nu/kT\right) - 1} \qquad (3.6)$$

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is simply the photonic black body spectrum. Equation (3.4) suffers from the embarrassing prediction that there should be an infinite amount of energy per unit volume, since

$$\lim_{\nu_c \to \infty} \int_0^{\nu_c} \rho(\nu, T) d\nu$$

is divergent. Indeed, writing

$$\rho(\nu, T) = \rho_{vac}(\nu) + \rho_{rad}(\nu, T), \tag{3.7}$$

it is easily seen that the divergence is a consequence of the temperature independent vacuum fluctuation term because

$$\int_0^{\infty} \rho_{rad}(\nu, T) d\nu = \frac{\pi^2 k^4}{15\hbar^3 c^3} T^4 \tag{3.8}$$

simply yields the customary Stefan-Boltzmann law. To circumvent this divergence, we suggested in [1] that Equation (3.5) is only valid up to a certain cutoff frequency ν_c so that the total energy associated with $\rho_{vac}(\nu)$ is given by

$$\int_0^{\nu_c} \rho_{vac}(\nu) d\nu = \frac{4\pi h}{c^3} \int_0^{\nu_c} \nu^3 d\nu = \frac{\pi h}{c^3} \nu_c^4. \tag{3.9}$$

We noted that a future experiment could examine whether the measured vacuum fluctuations in Fig. 1 of [1] might be a signature of dark energy. If so, one would expect to see a cutoff in the measured spectrum at

$$\nu_c \simeq (1.69 \pm 0.05) \times 10^{12} \quad \text{Hz}, \tag{3.10}$$

where this value of ν_c is obtained by setting

$$\frac{\pi h}{c^3} \nu_c^4 \simeq \rho_{dark} = (3.9 \pm 0.4) \text{GeV/m}^3 \tag{3.11}$$

(ρ_{dark} is the currently observed dark energy density in the universe [1]).

Jetzer and Straumann [5] have criticized the hypothesis of [1] based on two different points. In their own words:

- Point 1. " $\cdots$ the spectral density originally comes from a simple rational expression of Boltzmann factors, which are not related to zero-point energies."

  To illustrate their point, Jetzer and Straumann consider a simplified model of the van der Waals force between two harmonic oscillators and calculate the response of the system to distance changes. Their result is independent of zero-point energies of the two oscillators and from this they conclude that the same also holds for the measured spectrum (2.1) in Josephson junctions.

- Point 2. " ...the absolute value of the zero-point energy of a quantum mechanical system has no physical meaning when gravitational coupling is ignored. All that is measurable are changes of the zero-point energy under variations of system parameters or of external couplings, like an applied voltage."

  Based on this general statement, Jetzer and Straumann claim that experiments based on Josephson junctions are unable to detect dark energy since only differences in vacuum energy would be physically relevant.

Here we show in Section IV that Point 1 is misleading since the observed spectra in Josephson junctions have nothing to do with van der Waals forces. In Section V we argue that Point 2 is theoretically unclear (since the quantum noise in Josephson junctions has not been shown to be renormalizable) but experimentally testable.

IV. POINT 1

The justification of Point 1 of Jetzer and Straumann [5] is based on an equilibrium statistical mechanical model for the van der Waals interaction between two identical harmonic oscillators. The authors point out that a simple transformation can decouple the oscillators. The ground state of the decoupled system is the sum of the zero-point energies of the two decoupled oscillators and the corresponding van der Waals force is independent of the zero-point energies of the original oscillators.

Our response to Point 1 is based on the following four observations.

1. The simple model discussed in [5] is neither a valid description of the shunting resistor nor of the Josephson junction. Jetzer and Straumann make computations for van der Waals forces, whereas the measured spectra in the Josephson junctions are a consequence of a completely different effect, the ac Josephson effect [7]. Oversimplified *theoretical* models may not shed any light on the origin of *measured* noise spectra in Josephson junctions.

2. What is *measured* in the experiment of Koch et al. [2] is the spectrum of current fluctutions in the resistive shunt, mixed down at the Josephson frequency. The fact that the *experimental* data in [2] is so closely fit by Equation (2.1) is an indication that at low temperatures there is a significant correspondence between the behaviour of this superconducting device and the prediction of the corresponding theoretical treatment.

   Jetzer and Straumann claim, on the basis of their simplified model for van der Waals forces, that the linear term $\hbar\omega/2$ in Equation (2.1) cannot be due to vacuum fluctuations. It may be a matter of semantics to argue about what to call the source of this term, but their contention contradicts the received wisdom [6, 8, 9] which clearly singles out zero-point fluctuations as the source underlying the linear term $\hbar\omega/2$ in the spectrum.

3. Arguments for why vacuum (zero-point) fluctuations have a measurable effect in Josephson junctions have been given by various authors, e.g. [8]. Namely, a driven Josephson junction is a non-equilibrium system, and non-equilibrium systems can be influenced by vacuum fluctuations in a measurable way. For example, zero-point fluctuations can cause excited atoms to return to the ground state, thus producing an experimentally detectable effect. The argument against this observation presented in [5] is based on equilibrium statistical mechanics and does not incorporate non-equilibrium effects.

4. What is really at the root of the measured noise spectra in resistors is the fluctuation dissipation theorem [4, 9, 10] which precisely predicts a power spectrum as given by Equation (2.1). This spectrum has been experimentally confirmed by Koch et al.[2] up to frequencies of 0.6 THz. All textbooks [6, 9] and classical papers [4, 10] on the subject emphasize the fact that the linear term in the spectrum is induced by zero-point fluctuations.

Based on these points, we find Point 1 made by Jetzer and Straumannn to be unconvincing.

V. POINT 2

Turning to Point 2, it is clear that experiments involving van der Waals forces or the Casimir effect can only probe differences in vacuum energy. This is well known and related to the fact that QED is a renormalizable theory. Adding an arbitrary constant to the vacuum energy density leaves the physical predictions of this theory invariant. The correct conclusion is that experiments based on the Casimir effect have no chance of measuring the absolute value of vacuum energy.

The Josephson junction experiment, however, exploits a different effect which apparently has nothing to do with the Casimir effect. The theory of dissipative non-equilibrium quantum systems, such as driven Josephson junctions, is much less well understood than the Casimir effect. Whether the dissipative quantum theory underlying resistively shunted Josephson junctions can be renormalized is presently unclear. Hence the absolute value of vacuum energy may well have physical meaning for these kinds of superconducting quantum systems.

To illustrate this point, assume that only differences in vacuum energy are relevant for the Josephson junction experiment of Koch et al., as Point 2 suggests. It should then be possible to add an arbitrary constant (with the dimension of energy) to Equation (2.2), without changing the physical predictions of the theory. In our case the underlying theory is provided by the fluctuation dissipation theorem [4, 6, 9] which predicts in complete generality that the mean square fluctuations $\langle V^2 \rangle$ of the voltage in the shunting resistor are given by

$$< V^2 >= \frac{2}{\pi} \int \bar{U}(\omega/2\pi, T) R(\omega) d\omega \qquad (5.12)$$

where $\bar{U}(\nu, T)$ is given by Equation (2.2) and $R(\omega)$ is the shunting resistor. If we change $\bar{U}$ by an additive constant C to

$$\tilde{U}(\omega/2\pi, T) = \frac{1}{2}\hbar\omega + C + \frac{\hbar\omega}{exp(\hbar\omega/kT) - 1}, \qquad (5.13)$$

the result would contradict the results of the Koch et al. [2] experiment. Any $C \neq 0$ would imply voltage fluctuations in the resistor different from those actually measured.

We thus conclude that Point 2 is unclear from a theoretical point of view, and further that the resolution of this question cannot be decided on purely theoretical grounds. Rather, further experimental investigation is necessary. In [1] we suggested an experimental check to see whether a cutoff in the measurable spectrum could be observed near the critical frequency $\nu_c = 2\pi\omega_c = 1.7$ THz corresponding to dark energy density. If such a cutoff is observed, it would indeed be the *new physics* underlying this cutoff that makes the system couple to gravity and make the absolute value of vacuum energy physically relevant. Virtual photons that are not gravitationally active may well exist beyond this cutoff, it is just the gravitationally active part of vacuum fluctuations that would cease to exist at ν_c.

A repeat of the Koch experiment, based on new types of Josephson junctions operating in the THz region, will now be carried out by Warburton [11] and Barber and Blamire [12]. These new experiments will measure the noise spectrum up to frequencies exceeding the critical frequency $\nu_c = 1.7$ THz corresponding to dark energy density, using both nitride and cuprate based Josephson junctions. This is an interesting experimental project since the fluctuation dissipation theorem and its potential contribution to dark energy density has never been tested before at these high frequencies.

VI. CONCLUSIONS

It is the thesis of this note that the arguments of Jetzer and Straumann [5] against the hypothesis formulated in [1] are not applicable to our system. The arguments presented in [5] are based on a model for the van der Waals forces between two harmonic oscillators, which have nothing to do with the measured noise spectra in Josephson junctions. We further contend that the *only* way to really test the hypothesis that there is a cutoff in the frequency spectrum of measurable vacuum fluctuations is to actually do the experiment. Appeal to theoretical arguments extended to situations in which the theory has not been verified do not shed any light on the (so far unknown) nature of dark energy.

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