Chaotic scalar fields as models for dark energy

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Abstract

We consider stochastically quantized self-interacting scalar fields as suitable models to generate dark energy in the universe. Second quantization effects lead to new and unexpected phenomena if the self interaction strength is strong. The stochastically quantized dynamics can degenerate to a chaotic dynamics conjugated to a Bernoulli shift in fictitious time, and the right amount of vacuum energy density can be generated without fine tuning. It is numerically observed that the scalar field dynamics distinguishes fundamental parameters such as the electroweak and strong coupling constants as corresponding to local minima in the dark energy landscape. Chaotic fields can offer possible solutions to the cosmological coincidence problem, as well as to the problem of uniqueness of vacua.

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1 Introduction

There is by now convincing observational evidence that the universe is currently in a phase of accelerated expansion [1, 2]. The favored explanation for this behavior is the existence of vacuum energy or, in a more general setting, of dark energy. The observations suggest that the universe currently consist of approximately 73 % dark energy, 23 % dark matter, and 4% ordinary matter [3]. The nature and origin of the dominating dark energy component is not understood, and many different models co-exist. The simplest models associate dark energy with the vacuum energy of some unknown self-interacting scalar field, whose potential energy yields a cosmological constant [4]. In quintessence models slowly evolving scalar fields with a nontrivial equation of state are considered [5]. String theory also yields possible candidates of scalar fields who might generate dark energy, in form of run-away dilatons and moduli fields [6]. Various exotic forms of matter such as phantom matter [7] and Born-Infeld quantum condensates [8] are currently being discussed. For some superstring cosmology ideas related to small cosmological constants, see also [9].

When trying to formulate a suitable model for dark energy, at least two unsolved fundamental problems arise:

1. *The cosmological constant problem.* Why is the observed vacuum energy density so small, as compared to typical predictions of particle physics models? From electroweak symmetry breaking via the Higgs mechanism one obtains a vacuum energy density prediction that is too large by a factor 10^{55} as compared to the currently observed value. Spontaneous symmetry breaking in GUT models is even worse, it yields a discrepancy by a factor 10^{111}.

2. *The cosmological coincidence problem.* Why is the order of magnitude of the currently observed vacuum energy density the same as that of the matter density? A true cosmological constant stays constant during the expansion of the universe, whereas the matter energy density decreases with a^{-3}, where $a(t)$ is the scale factor in the Robertson Walker metric. It looks like a very strange coincidence that right now we live at an epoch where the vacuum energy density and matter density have the same order of magnitude, if during the evolution of the universe one is constant and the other one decreases as $a(t)^{-3}$.

To this list one may add yet another fundamental problem, which we may call

3. *The uniqueness problem.* String theory allows for an enourmous amount of possible vacua after compactification. In each of these states the fundamental constants of nature can take on different possible values. But what is the mechanism that selects out of these infinitely many possibilities the physically relevant vacuum state, with its associated fundamental constants that give rise to a universe of the type we know it (that ultimately even enabled the development of life)? Relating the answer purely to an anthropic principle seems unsatisfactory.

In this paper we consider a new model for dark energy which, as compared to other models, is rather conservative. It just associates dark energy with self-interacting scalar fields corresponding to a φ^4-theory, which is second quantized. However, the fundamental difference to previous approaches is that these fields are very strongly (rather than weakly) self-interacting, and that 2nd quantization effects play an important role. We will use as the relevant method to quantize the scalar fields the stochastic quantization method introduced by Parisi and Wu [10]. In the fictitious time variable of this approach, the fields will turn out to perform rapid deterministic chaotic oscillations, due to the fact that we consider not a weakly but a very strongly self-interacting field. This chaotic behavior is a new effect not present in any classical treatment. It is generally well known that chaos plays an important role in general relativity [11], quantum field theories [12, 13, 14], and string theories [15]. The main result of our consideration is that the chaotic field theories considered naturally generate a small cosmological constant and have the scope to offer simultaneous solutions to the cosmological coincidence and uniqueness problem.

Our physical interpretation is to associate the chaotic behaviour of the scalar fields with tiny vacuum fluctuations which are allowed within the bounds set by the uncertainty relation, due to the finite age of the universe. This interpretation naturally leads to the right amount of dark energy density being generated, and fine tuning can be avoided. The chaotic fields have a classical equation of state close to $w = -1$, and can thus account for the accelerated expansion of the universe. However, since they are interpreted in terms of vacuum fluctuations, energy is not conserved for them. This property will help to avoid the cosmological coincidence problem.

The chaotic model also contains an interesting symmetry between gravitational and gauge couplings. In our model the role of a metric for the 5th coordinate (the fictitious time) is taken over by dimensionless coupling constants which are given by the ratio of the fictious time lattice constant and physical time lattice constant squared (both lattice constants can still go to zero, just their ratio is fixed). These coupling constants do not occur in any classical treatment but are entirely a consequence of our second quantized treatment. The vacuum energy generated depends on these couplings in a non-trivial way. The physical significance of our model is illustrated by the fact that we numerically observe the vacuum energy to have local minima for coupling constants that numerically coincide with running electroweak coupling strengths, evaluated at the known fermionic mass scales, as well as running strong coupling constants evaluated at the known bosonic mass scales. This numerical observation, previously reported in [14], is now embedded into a cosmological context. The role of the chaotic fields in the universe can be understood in the sense that they are responsible for fixing and stabilizing fundamental parameters as local minima in the dark energy landscape. This is somewhat similar to the role the dilaton field plays in string theory after supersymmetry breaking.

Our numerical discovery of local minima that coincide with known standard model coupling constants makes it very unlikely that there are different universes with different fundamental parameters. In fact, the numerical results provide strong evidence that there is a unique vacuum state of the universe that possesses minimum vacuum energy precisely for the known set of standard model parameters.

This paper is organized as follows. In section 2 we show how a second-quantized scalar field dynamics can degenerate to a chaotic dynamics in fictitious time. Our main example is a chaotic φ^4-theory leading to 3rd order Tchebyscheff maps, which is dealt with in section 3. In section 4 we present a physical interpretation of the chaotic dynamics using the uncertainty relation, which in a natural way fixes the order of magnitude of the vacuum energy density to be generated. Section 5 deals with energy, pressure and classical equation of state of the chaotic fields. In section 6 we consider the Einstein equations associated with our model and discuss a possible way to avoid the cosmological coincidence problem. Section 7 yields a prediction for the current ratio of matter energy density and dark energy density to the critical energy density. Finally, in section 8 we describe how local minima of the dark energy landscape generated by the chaotic fields can fix the fundamental parameters.

2 Stochastic quantization of strongly self-interacting scalar fields

Let us consider a self-interacting scalar field φ in Robertson-Walker metric. For a complete theory describing all quantum mechanical fluctuations we need to second-quantize it. This can be done via stochastic quantization. In the Parisi-Wu approach of stochastic quantization one considers a stochastic differential equation evolving in a fictitious time variable s, the drift term being given by the classical field equation [10]. Quantum mechanical expectations correspond to expectations with respect to the generated stochastic processes in the limit $s \to \infty$. The fictitious time s is different from the physical time t, it is just a helpful fifth coordinate to do 2nd quantization. Neglecting spatial gradients the field φ is a function of physical time t and fictitious time s. The 2nd quantized equation of motion is

$$\frac{\partial}{\partial s}\varphi = \ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) + L(s,t), \tag{1}$$

where H is the Hubble parameter, V is the potential under consideration and $L(s,t)$ is Gaussian white noise, δ-correlated both in s and t. For e.g. a numerical simulation we may discretize eq. (1) using

$$s = n\tau \tag{2}$$

$$t = i\delta, \tag{3}$$

where n and i are integers and τ is a fictitious time lattice constant, δ is a physical time lattice constant. The continuum limit requires $\tau \to 0$, $\delta \to 0$, but we will later argue that it makes physical sense to keep small but finite lattice constants of the order of the Planck length. We obtain

$$\frac{\varphi^i_{n+1} - \varphi^i_n}{\tau} = \frac{1}{\delta^2}(\varphi^{i+1}_n - 2\varphi^i_n + \varphi^{i-1}_n) + 3\frac{H}{\delta}(\varphi^i_n - \varphi^{i-1}_n) + V'(\varphi^i_n) + noise \tag{4}$$

This can be written as the following recurrence relation for the field φ^i_n

$$\varphi^i_{n+1} = (1-\alpha)\left\{\varphi^i_n + \frac{\tau}{1-\alpha}V'(\varphi^i_n)\right\} + 3\frac{H\tau}{\delta}(\varphi^i_n - \varphi^{i-1}_n) + \frac{\alpha}{2}(\varphi^{i+1}_n + \varphi^{i-1}_n) + \tau\cdot noise, \tag{5}$$

where a dimensionless coupling constant α is introduced as

$$\alpha := \frac{2\tau}{\delta^2}. \tag{6}$$

We also introduce a dimensionless field variable Φ_n^i by writing $\varphi_n^i = \Phi_n^i p_{max}$, where p_{max} is some (so far) arbitrary energy scale. The above scalar field dynamics is equivalent to a spatially extended dynamical system (a coupled map lattice [16]) of the form

$$\Phi_{n+1}^i = (1-\alpha)T(\Phi_n^i) + \frac{3}{2}H\delta\alpha(\Phi_n^i - \Phi_n^{i-1}) + \frac{\alpha}{2}(\Phi_n^{i+1} + \Phi_n^{i-1}) + \tau \cdot noise, \quad (7)$$

where the local map T is given by

$$T(\Phi) = \Phi + \frac{\tau}{p_{max}(1-a)} V'(p_{max}\Phi). \quad (8)$$

Here the prime means

$$' = \frac{\partial}{\partial\varphi} = \frac{1}{p_{max}}\frac{\partial}{\partial\Phi}. \quad (9)$$

Note that a symmetric diffusively coupled map lattice of the form

$$\Phi_{n+1}^i = (1-\alpha)T(\Phi_n^i) + \frac{\alpha}{2}(\Phi_n^{i+1} + \Phi_n^{i-1}) + \tau \cdot noise \quad (10)$$

is obtained if $H\delta << 1$, equivalent to

$$\delta << H^{-1}, \quad (11)$$

meaning that the physical time lattice constant δ is much smaller than the age of the universe. In this case the term proportional to H in eq. (7) can be neglected. The local map T depends on the potential under consideration. Since we restrict ourselves to real scalar fields φ, T is a 1-dimensional map.

The main result of our consideration is that iteration of a coupled map lattice of the form (10) with a given map T has physical meaning: It means that one is considering the second-quantized dynamics of a self-interacting real scalar field φ with a force V' given by

$$V'(\varphi) = \frac{1-\alpha}{\tau}\left\{-\varphi + p_{max}T\left(\frac{\varphi}{p_{max}}\right)\right\}. \quad (12)$$

Integration yields

$$V(\varphi) = \frac{1-\alpha}{\tau}\left\{-\frac{1}{2}\varphi^2 + p_{max}\int d\varphi\, T\left(\frac{\varphi}{p_{max}}\right)\right\} + const. \quad (13)$$

In terms of the dimensionless field Φ this can be written as

$$V(\varphi) = \frac{1-\alpha}{\tau}p_{max}^2\left\{-\frac{1}{2}\Phi^2 + \int d\Phi T(\Phi)\right\} + const. \quad (14)$$

3 Chaotic φ^4-theory

An interesting observation is the following one. The lattice constant τ of fictitious time should be small, in order to approximate the continuum theory, which is ordinary quantum field theory. If τ is small one naively expects the map T given by eq. (8) to be close to the identity for finite forces V', since $\tau V'/p_{max}$ is small. What about, however, very strong forces V' due to very strongly self-interacting fields? If p_{max}/τ is of the same order of magnitude as V' then a nontrivial map T can arise. In particular, this map may even exhibit chaotic behaviour.

As a distinguished example of a φ^4-theory generating strongest possible chaotic behaviour, let us consider the map

$$\Phi_{n+1} = T_{-3}(\Phi_n) = -4\Phi_n^3 + 3\Phi_n \tag{15}$$

on the interval $\Phi \in [-1, 1]$. T_{-3} is the negative third-order Tchebyscheff map, a standard example of a map exhibiting strongly chaotic behaviour. The corresponding potential is given by

$$V_{-3}(\varphi) = \frac{1-\alpha}{\tau}\left\{\varphi^2 - \frac{1}{p_{max}^2}\varphi^4\right\} + const, \tag{16}$$

or, in terms of the dimensionless field Φ,

$$V_{-3}(\varphi) = \frac{1-\alpha}{\tau} p_{max}^2 (\Phi^2 - \Phi^4) + const. \tag{17}$$

Apparently, starting from this potential we obtain by second quantization a field φ that rapidly fluctuates in fictitious time on some finite interval, provided that initially $\varphi_0 \in [-p_{max}, p_{max}]$. The small noise term in eq. (10) can be neglected as compared to the deterministic chaotic fluctuations of the field.

Of physical relevance are the expectations of suitable observables with respect to the ergodic chaotic dynamics. For example, the expectation $\langle V_{-3}(\varphi)\rangle$ of the potential is a possible candidate for vacuum energy in our universe. One obtains

$$\langle V_{-3}(\varphi)\rangle = \frac{1-\alpha}{\tau} p_{max}^2 (\langle\Phi^2\rangle - \langle\Phi^4\rangle) + const. \tag{18}$$

For uncoupled Tchebyscheff maps ($\alpha = 0$), expectations of any observable A can be evaluated as the ergodic average

$$\langle A \rangle = \int_{-1}^{+1} A(\Phi) d\mu(\Phi), \tag{19}$$

with the natural invariant measure being given by

$$d\mu(\Phi) = \frac{d\Phi}{\pi\sqrt{1-\Phi^2}} \tag{20}$$

(see any textbook on chaotic dynamics, e.g. [17]). This measure describes the probability distribution of the iterates under long-term iteration. From eq. (20) one obtains $\langle \Phi^2 \rangle = \frac{1}{2}$ and $\langle \Phi^4 \rangle = \frac{3}{8}$, thus

$$\langle V_{-3}(\varphi) \rangle = \frac{1}{8}\frac{p_{max}^2}{\tau} + const. \tag{21}$$

Alternatively, we may consider the positive Tchebyscheff map $T_3(\Phi) = 4\Phi^3 - 3\Phi$. This basically exhibits the same dynamics as T_{-3}, up to a sign. Repeating the same calculation we obtain

$$V_3(\varphi) = \frac{1-\alpha}{\tau}\left\{-2\varphi^2 + \frac{1}{p_{max}^2}\varphi^4\right\} + const \tag{22}$$

and

$$V_3(\varphi) = \frac{1-\alpha}{\tau} p_{max}^2 (-2\Phi^2 + \Phi^4). \tag{23}$$

For the expectation of the vacuum energy one gets

$$\langle V_3(\varphi) \rangle = \frac{1-a}{\tau} p_{max}^2 (-2\langle \Phi^2 \rangle + \langle \Phi^4 \rangle) + const, \tag{24}$$

which for $\alpha = 0$ reduces to

$$\langle V_3(\varphi) \rangle = -\frac{5}{8}\frac{p_{max}^2}{\tau} + const. \tag{25}$$

Symmetry considerations between T_{-3} and T_3 suggest to take the additive constant *const* as

$$const = +\frac{1-\alpha}{\tau} p_{max}^2 \frac{1}{2}\langle \Phi^2 \rangle. \tag{26}$$

In this case one obtains the fully symmetric equation

$$\langle V_{\pm 3}(\varphi)\rangle = \pm\frac{1-\alpha}{\tau}p_{max}^2\left\{-\frac{3}{2}\langle\Phi^2\rangle + \langle\Phi^4\rangle\right\}, \tag{27}$$

which for $\alpha \to 0$ reduces to

$$\langle V_{\pm 3}(\varphi)\rangle = \pm\frac{p_{max}^2}{\tau}\left(-\frac{3}{8}\right). \tag{28}$$

The simplest model for dark energy in the universe, as generated b a chaotic φ^4-theory, would be to identify $\frac{3}{8}p_{max}^2/\tau = \rho_\Lambda$, the constant vacuum energy density corresponding to a classical cosmological constant Λ, which stays constant during the expansion of the universe. This is certainly a possible simple model. On the other hand, such an approach would neither solve the cosmological constant nor the cosmological coincidence problem. For this reason we turn to a more abvanced model in the following, which naturally produces the right amount of vacuum energy density in the universe.

4 Reproducing the currently measured dark energy density

To obtain quantitative statements on the dark energy density as generated by some chaotically evolving field φ, let us fix the free parameters τ and p_{max} by some physical arguments. Let us start with the parameter τ. It is the lattice constant of fictitious time t and has dimension GeV^{-2}. Ordinary stochastic quantization based on Gaussian white noise requires the continuum limit $\tau \to 0$. But since quantum field theory runs into difficulties at the Planck scale m_{Pl} and is expected to be replaced by a more advanced theory at this scale, it is most reasonable to take the small but finite value

$$\tau \sim m_{Pl}^{-2}. \tag{29}$$

Next, consider the parameter p_{max}. It has dimension GeV and describes the maximum energy scale of our rapidly fluctuating scalar fields φ, who take on values on the finite interval $[-p_{max}, p_{max}]$. A natural value of p_{max} follows if one associates the rapidly fluctuating chaotic fields φ_n^i with vacuum fluctuations that are allowed due to the uncertainty relation

$$\Delta E \Delta t = O(\hbar). \tag{30}$$

Taking $\Delta t \sim t$ of the order of the age of the universe, a corresponding energy uncertainty ΔE arises. This ΔE is very large for a very young universe, and then decreases to extremely small values for the current age of the universe of about 13.7 Gyr. Any finite age t of the universe implies that spontaneous vacuum fluctuations with energies of order $\Delta E \sim t^{-1}$ can occur. It is physically plausible to identify these energy fluctuations ΔE with the rapidly fluctuating chaotic fields $\varphi = p_{max}\Phi_n^i$, since both ΔE and φ live on a finite interval, and both fluctuate in an unpredictable way. The uncertainty relation (30) together with $\Delta t \sim t$ implies (in units where $\hbar = c = 1$)

$$p_{max} \sim \frac{1}{t}. \tag{31}$$

In this way the energy scale p_{max} occurring in the chaotic field theories is most naturally identified with the inverse age of the universe. However, note that this quantum mechanical interpretation in terms of vacuum fluctuations requires a *chaotic* map T, since some regularly evolving Φ_n^i cannot be associated with fluctuations at all.

It is remarkable that by taking eq. (29) and eq. (31) together, the right amount of vacuum energy follows without any fine tuning. One has for generic chaotic maps T

$$\langle V(\varphi)\rangle \sim \frac{p_{max}^2}{\tau} \sim H^2 m_{Pl}^2. \tag{32}$$

since $t^{-1} \sim H$. Moreover,

$$H^2 = \frac{8\pi G}{3}\rho_c \sim \frac{1}{m_{Pl}^2}\rho_c \tag{33}$$

where ρ_c denotes the critical density of a flat universe and $G = m_{Pl}^{-2}$ is the gravitational constant. Combining eq. (32) and (33) one obtains

$$\langle V(\varphi)\rangle \sim \rho_c, \tag{34}$$

as required and confirmed by current astronomical observations. Our simple physical interpretation, namely to interpret the chaotic fluctuations as vacuum fluctuations allowed due to the finite age of the universe, thus yields the right order of magnitude of the vacuum energy. Note that in this way the cosmological constant problem is avoided: The right order of magnitude of the vacuum energy density comes out with very natural assumptions, based on the uncertainty relation. No fine tuning is required.

Let us here produce some concrete numbers. The current age of the universe is $t_0 = (13.7 \pm 0.2)$ Gyr $= (4.32 \pm 0.06) \cdot 10^{17} s$ [3]. Using an uncertainty relation of the form $\Delta E \Delta t = \hbar/2$ we get $p_{max} = 1/(2t_0) = (7.62 \pm 0.08) \cdot 10^{-43}$ GeV. Choosing $\tau = \kappa m_{Pl}^{-2}$, where κ is some dimensionless number of $O(1)$, we get

$$\langle V(\varphi) \rangle = \frac{3}{8} \frac{p_{max}^2}{\tau} = (3.19 \pm 0.05) \cdot 10^{-47} \kappa^{-1} \; GeV^4 \tag{35}$$

The current observational estimate of dark energy density in the universe is [3]

$$\rho_\varphi^{Obs} = (2.9 \pm 0.3) \cdot 10^{-47} GeV^4, \tag{36}$$

which is consistent with $\kappa \approx 1$. If the observed dark energy in the universe is produced by our chaotic theory, then the measured data imply

$$\kappa = 1.10 \pm 0.10. \tag{37}$$

5 Energy density, pressure, and equation of state

The kinetic energy term of our chaotic fields is given by

$$T_{kin} = \frac{1}{2} \left(\frac{\partial}{\partial t} \varphi \right)^2 . \tag{38}$$

Discretized with lattice constant δ we obtain for the expectation of T_{kin}

$$\begin{aligned} \langle T_{kin} \rangle &= \frac{1}{2} \frac{p_{max}^2}{\delta^2} \langle (\Phi_n^i - \Phi_n^{i-1})^2 \rangle \\ &= \frac{p_{max}^2}{\tau} \frac{1}{2} \alpha (\langle \Phi^2 \rangle - \langle \Phi_n^i \Phi_n^{i-1} \rangle) \end{aligned} \tag{39}$$

In particular, for $\alpha \to 0$ the expectation of kinetic energy vanishes, and a universe mainly filled with such a field is vacuum energy dominated.

In general, the expectation of the total energy density $\langle \rho \rangle$ is given by

$$\langle \rho \rangle = \langle T_{kin} \rangle + \langle V \rangle \tag{40}$$

and the expectation of the pressure by

$$\langle p\rangle = \langle T_{kin}\rangle - \langle V\rangle. \tag{41}$$

For the map T_{-3} one obtains

$$\langle\rho\rangle = \frac{p_{max}^2}{\tau}\left\{\frac{\alpha}{2}(\langle\Phi^2\rangle - \langle\Phi_n^i\Phi_n^{i-1}\rangle) + (1-\alpha)(\frac{3}{2}\langle\Phi^2\rangle - \langle\Phi^4\rangle)\right\} \tag{42}$$

$$\langle p\rangle = \frac{p_{max}^2}{\tau}\left\{\frac{\alpha}{2}(\langle\Phi^2\rangle - \langle\Phi_n^i\Phi_n^{i-1}\rangle) - (1-\alpha)(\frac{3}{2}\langle\Phi^2\rangle - \langle\Phi^4\rangle)\right\}, \tag{43}$$

where the additive constant of the self-interacting potential is fixed by the symmetry consideration of section 3. The above equations yield the equation of state

$$w = \frac{\langle p\rangle}{\langle\rho\rangle} \tag{44}$$

which varies as a function of the coupling α in a nontrivial way.

For $\alpha = 0$, the equation of state of our fields is $w = -1$, since the expectation of kinetic energy vanishes (all expectations should be interpreted as quantum mechanical expectations with respect to second quantization). For small α, w is close to -1. It should be clear that although our fields fluctuate rapidly in both physical and fictitious time, these fluctuations are averaged away when doing the quantum mechanical expectations. Thus the classical picture that arises out of this 2nd quantized rapidly fluctuating model is a very homogeneous field.

The expectations in eqs. (42), (43) are easily numerically calculated by long-term iterating the coupled map lattice for random initial conditions and averaging over all i and n. We used lattices of size 10000 with periodic boundary conditions. The result for the equation of state $w(\alpha)$ is displayed in Fig. 1. For small α, w grows approximately in a linear way. It monotonously increases from $w = -1$ for $\alpha = 0$ to $w = +1$ for $\alpha = 1$, up to a wiggle at $\alpha \approx 0.12$. Fig. 2 shows the corresponding (classical) energy density and pressure of the field. To account for the currently observed dark energy in the universe, most chaotic fields must have a coupling α smaller than about 0.04. Larger α are ruled out by the observations providing evidence for $w < -0.75$ [3].

6 Einstein equations and dynamical evolution

Let us now consider the Einstein equations for a homogeneous and isotropic universe that consists of three different components: matter, radiation, and chaotic fields. These three components are labeled by the indices m, r, φ, respectively. One has

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G(\rho_\varphi + \rho_m + \rho_r) \tag{45}$$

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho_\varphi + 3p_\varphi + \rho_m + \rho_r + 3p_r). \tag{46}$$

Here ρ_j denotes the (classical) energy density of component j, and p_j the pressure. For simpler notation we omit the expectation values $\langle \cdots \rangle$. The equation of state of each component is $w_j = p_j/\rho_j$. As it is well known, one has for matter $w_m = 0$, for radiation $w_r = 1/3$, whereas the equation of state w_φ of the chaotic fields φ depends on the coupling α (see Fig. 1).

The Einstein equations are usually supplemented by the assumption of conservation of energy for each species j,

$$\dot{\rho}_j = -3H(\rho_j + p_j). \tag{47}$$

These equations can be derived from the Einstein equations under the additional assumption of adiabatic expansion, i.e. one assumes that no entropy is produced.

For a universe dominated by a species j with constant equation of state $w_j = p_j/\rho_j$ eq. (47) leads to

$$\rho_j \sim a^{-3(1+w_j)}. \tag{48}$$

We obtain the well-known result that for matter $\rho_m \sim a^{-3}$, for radiation $\rho_r \sim a^{-4}$, whereas for true classical vacuum energy (a cosmological constant Λ) with $w = -1$ one has no dependence on a at all, $\rho_\Lambda = const.$

While the assumption of energy conservation (or adiabatic expansion) is certainly reasonable for matter and radiation, for our chaotic fields it is not. In fact, the parameters of the self-interacting potential are explicitly time-dependent, hence the assumption of energy conservation is obviously unjustified. We have

$$\rho_\varphi = \langle V_{-3}(\varphi)\rangle \sim \frac{p_{max}^2}{\tau} \sim \frac{m_{Pl}^2}{t^2}. \tag{49}$$

The physical interpretation for this time-dependence is quite clear. A very small time interval t implies large fluctuating energies ΔE, whereas a large time interval t implies very small fluctuating ΔE, a consequence of the uncertainty relation. It is thus wrong to assume energy conservation for the chaotic fields. These fields model vacuum fluctuations, and by definition vacuum fluctuations violate energy conservation. Moreover, these fields (as any chaotic map) constantly produce entropy, hence the assumption of an adiabatic expansion, which would lead to eq. (48), is incorrect either.

As a consequence, eq. (48) is valid for $j = r$ and $j = m$ but not for $j = \varphi$. The correct time-dependence of ρ_φ is given by eq. (49). Suppose the universe surrounding the chaotic fields is dominated by a species with equation of state $w > -1$ (typically matter or radiation), then

$$a(t) \sim t^{\frac{2}{3(1+w)}} \Longleftrightarrow t^{-2} \sim a^{-3(1+w)}. \tag{50}$$

Putting this into eq. (49) we obtain

$$\rho_\varphi = \langle V_{-3}(\varphi) \rangle \sim a^{-3(1+w)}, \tag{51}$$

i.e. the vacuum energy density associated with the chaotic field decays *in the same way* with a as the density of the dominating species.

This property can help to naturally avoid the cosmological coincidence problem. Consider e.g. the following scenario. Initially (say, shortly after inflation) we may have a state where $\rho_r \sim \rho_\varphi >> \rho_m$. Then ρ_φ first decays approximately as a^{-4}, since the universe is radiation dominated. At some stage we arrive at $\rho_r \sim \rho_\varphi \sim \rho_m$, and from then on matter dominates over radiation, so that from then on ρ_φ decays approximately as a^{-3}. During the late-time evolution of the universe, ρ_φ will always stay of the same order of magnitude as ρ_m, since both ρ_φ and ρ_m decay as a^{-3}. Hence the cosmological coincidence problem is avoided in a natural way. In spite of this, for small enough couplings α the chaotic fields have a classical equation of state close to $w_\varphi = -1$, and can thus produce the accelerated expansion of the universe via eq. (46).

What is our physical interpretation of the chaotic fields in the late-time universe? For $\alpha = 0$, it can be rigorously proved that rescaled deterministic chaotic Tchebyscheff maps can be used to generate spatio-temporal Gaussian white noise on a larger scale [12, 13]. In other words, on fictitious time scales $\tau' >> \tau$ and physical time scales $\delta' >> \delta$ the chaotic noise just looks like ordinary Gaussian white noise. We may thus couple the chaotic fields φ

to ordinary standard model fields in order to second quantize the standard model fields, i.e. use the chaotic fields as a source of quantization noise. This is the basic idea of the so-called 'chaotic quantization' approach [12]. The chaotic fields are well embedded in this way and since they are just playing the role of quantization noise, we do not expect them to have any disturbing influence on, say, baryogenesis and similar processes in the early universe. In this interpretation dark energy just arises out of the expectation of a classical potential that generates quantization noise.

7 Prediction of Ω_φ and Ω_m

Our approach allows for the prediction of the order of magnitude of cosmological parameters such as $\Omega_\varphi = \rho_\varphi/\rho_c$ and $\Omega_m = \rho_m/\rho_c$ at the present time. Let us start from the uncertainty relation in the form

$$\Delta E \Delta t = \frac{\hbar}{2}, \tag{52}$$

which implies

$$p_{max} = \frac{1}{2t}. \tag{53}$$

Choosing the time scale $\tau = \kappa m_{Pl}^{-2}$ we get for $\alpha \approx 0$

$$\langle V_{-3}(\varphi) \rangle = \frac{3}{8} \frac{p_{max}^2}{\tau} = \frac{3}{32} \frac{m_{Pl}^2}{\kappa} \frac{1}{t^2} \tag{54}$$

During the radiation dominated period of the universe one has for the energy density of radiation

$$\rho_r = \frac{\pi^2}{30} N(T) T^4, \tag{55}$$

where T is the temperature and $N(T)$ is the number of relativistic particle degrees of freedom. There is also a relation between time and temperature, namely

$$t = \sqrt{\frac{90}{32\pi^3 N(T)}} \frac{m_{Pl}}{T^2}. \tag{56}$$

Putting eq. (56) into (54) one obtains

$$\langle V_{-3}(\varphi) \rangle = \rho_\varphi = \frac{1}{30} \pi^3 \frac{1}{\kappa} N(T) T^4 = \frac{\pi}{\kappa} \rho_r. \tag{57}$$

This equation once again shows that it is reasonable to assume that ρ_r and ρ_φ have the same order of magnitude. Since ρ_φ decays in the same way as ρ_r, eq. (57) is valid during the entire radiation dominated epoch. Finally ρ_r falls below ρ_m and from then on we have

$$\rho_\varphi \approx \frac{\pi}{\kappa}\rho_m. \tag{58}$$

This implies a prediction for $\Omega_\varphi := \rho_\varphi/\rho_c$ at the present time, namely

$$\Omega_\varphi = \frac{\rho_\varphi}{\rho_\varphi + \rho_m + \rho_r} \approx \frac{\pi/\kappa}{1+\pi/\kappa}, \tag{59}$$

neglecting ρ_r at the present epoch. In section 4 we saw that the currently observed dark energy density is best fitted by the value $\kappa = 1.10 \pm 0.10$. Eq. (59) yields with this value the prediction

$$\Omega_\varphi \approx 0.74 \quad \Omega_m \approx 0.26, \tag{60}$$

which is consistent with observations[3].

8 Fixing fundamental parameters

We have seen that chaotic fields generate the right amount of vacuum energy and have the scope to avoid the cosmological constant and coincidence problem. We now show that they also offer solutions to the problem of uniqueness of vacua.

First, let us slightly generalize the chaotic field dynamics (10) to

$$\Phi^i_{n+1} = (1-\alpha)T(\Phi^i_n) + \sigma\frac{\alpha}{2}(T^b(\Phi^{i-1}_n) + T^b(\Phi^{i+1}_n)) \tag{61}$$

(we neglect the small noise term). The case $\sigma = +1$ is called 'diffusive coupling', the case $\sigma = -1$ 'anti-diffusive coupling'. Chaotic fields with $b = 1$ are called to be of 'type A' ($T^1(\Phi) =: T(\Phi)$), chaotic fields with $b = 0$ to be of 'type B' ($T^0(\Phi) =: \Phi$). In [14] the chaotic fields were called 'chaotic strings', but this is only a different name for the same dynamics. Our derivation in section 2 lead to chaotic fields of B-type with diffusive coupling, but from a dynamical systems point of view all 4 degrees of freedom ($b = 0, 1,\ \sigma = \pm 1$) exist and are of relevance. As shown in detail in [14], there are two different types of vacuum energies for the chaotic fields, namely

1. the self energy

$$V(\alpha) := \frac{p_{max}^2}{\tau}\left(\frac{3}{2}\langle\Phi^2\rangle - \langle\Phi^4\rangle\right) \tag{62}$$

and

2. the interaction energy

$$W(\alpha) := \frac{p_{max}^2}{2\tau}\langle\Phi_n^i\Phi_n^{i+1}\rangle. \tag{63}$$

Basically, the self energy is the expectation of the potential that generates the chaotic dynamics in fictitious time, and the interaction energy is the expectation of the potential that generates the diffusive coupling in physical time. One may also define a total vacuum energy as $H^{\pm}(\alpha) := V(\alpha) \pm \alpha W(\alpha)$, where the $-$ sign corresponds to diffusive and the $+$ sign to anti-diffusive coupling. All additive constants are fixed by the postulate of invariance of the theory under global and local Z_2-transformations [13]. For small α the interaction energy can be neglected as compared to the self energy, moreover, the type-A and type-B forms are observed to have the same self energy in this limit.

The central hypothesis of this paper is a symmetry between standard model coupling constants and the chaotic field couplings α. We assume that for any dimensionless coupling constant α that appears in the standard model of electroweak and strong interactions, there is a corresponding chaotic field that is just coupled with this α. The universe then tries to reach a state of minimum vacuum energy by adjusting its free parameters in such a way that the chaotic fields reach a state of minimum vacuum energy.

While at first sight this may look like a purely theoretical concept, there is numerical evidence that this principle is indeed physically realized. As an example, Fig. 3 shows the self energy $V(\alpha) = \langle V_{-3}(\varphi)\rangle$ of our chaotic fields of type A with diffusive coupling in the low-coupling region. We observe that $V(a)$ has local minima at

$$a_1 = 0.000246(2) \tag{64}$$

$$a_2 = 0.00102(1) \tag{65}$$

$$a_3 = 0.00220(1) \tag{66}$$

(a_1 and a_3 are actually small local minima on top of the hill).

On the other hand, in the standard model of electroweak interactions the weak coupling constant is given by

$$\alpha_{weak} = \alpha_{el} \frac{(T_3 - Q \sin^2 \theta_W)^2}{\sin^2 \theta_W \cos^2 \theta_W} \tag{67}$$

Here Q is the electric charge of the particle ($Q = -1$ for electrons, $Q = 2/3$ for u-like quarks, $Q = -1/3$ for d-like quarks), and T_3 is the third component of the isospin ($T_3 = 0$ for right-handed particles, $T_3 = -\frac{1}{2}$ for e_L and d_L, $T_3 = +\frac{1}{2}$ for ν_L and u_L). Consider right-handed fermions f_R. With $\sin^2 \theta_W = \bar{s}_l^2 = 0.2318$ (as experimentally measured [18]) and the running electric coupling $\alpha_{el}(E)$ taken at energy scale $E = 3m_f$ we obtain from eq. (67) the numerical values

$$\begin{aligned} \alpha_{weak}^{d_R}(3m_d) &= 0.000246 & (68)\\ \alpha_{weak}^{c_R}(3m_c) &= 0.001013 & (69)\\ \alpha_{weak}^{e_R}(3m_e) &= 0.00220. & (70) \end{aligned}$$

There is an amazing numerical coincidence between the local minima a_1, a_2, a_3 of $V(\alpha)$ and the weak coupling constants of $f_R = u_R, c_R, e_R$, respectively.

Now regard the fine structure constant α_{el} and the Weinberg angle $\sin^2 \theta_W$ as a priori free parameters. Suppose these parameters would change to slightly different values. Then immediately this would produce larger vacuum energy $V(\alpha)$, since we get out of the local minima. The system is expected to be driven back to the local minima, and the fundamental parameters are stabilized in this way[2].

The above example is only one example of a large number of numerical coincidences observed. In [13, 14] an extensive numerical investigation of self energies, interaction energies, and total vacuum energies was performed for the above chaotic field theories. A large number of amazing numerical coincidences was found[3]. These results are described in detail in [13, 14], we here only summarize the main results.

1. The smallest (stable) zeros of the interaction energy $W(\alpha)$ coincide with running electroweak coupling constants, evaluated at energies given

[2]Note that the initial transient state of the universe until the minima are reached might well allow for a varying fine structure constant [19].

[3]For the φ^4-theory the relevant energy scale is always $E = 3m_f$. The factor 3 can be related to the index of the Tchebyscheff polynomial considered [13].

by the smallest fermionic mass scales. Type (A) describes d-quarks and electrons interacting electrically, type (B) u-quarks and neutrinos interacting weakly.

2. Local minima of the self energy $V(\alpha)$ coincide with running weak coupling constants of right-handed fermions, evaluated at the lightest fermionic mass scales.
3. Local minima of the total vacuum energy $H^+(\alpha)$ occur at running strong coupling constants evaluated at the lightest baryonic energy scales.
4. Local minima of the total vacuum energy $H^-(\alpha)$ occur at running strong couplings evaluated at the lightest mesonic energy scales.

In [13, 14] also chaotic fields corresponding to 2nd order Tchebyscheff maps were investigated (see Appendix), and the following numerical coincidences were found:

1. The smallest (stable) zeros of $W(\alpha)$ coincide with running strong coupling constants evaluated at the smallest bosonic mass scales. Type (A) describes the W boson, type (B) the Higgs boson.
2. Local minima of the self energy $V(\alpha)$ coincide with Yukawa and gravitational couplings evaluated at the fermionic mass scales.

For more details, see [13, 14].

All these numerically observed coincidences are not explainable as a random coincidence. Rather, they suggest to interpret the coupling constant α of our second-quantized chaotic field φ as a running gauge coupling. We are free to identify $\alpha = 2\tau/\delta^2$ with a gauge coupling, since the occurence of a ratio of lattice constants τ and δ^2 is a new effect in our 2nd quantized discretized theory, and there is no theory of this dimensionless number so far, which represents a kind of metric for the 5th coordinate (the fictitious time). So we are indeed free to make the hypothesis that α coincides with a running gauge coupling. By doing so, we implicitly construct a symmetry between gauge couplings and gravitational couplings, since usually the strength of the kinetic term in the action of a field is determined by the metric, i.e. gravitational effects, whereas here it is fixed by standard model coupling strengths. The chaotic fields appear to select out of the infinitely many vacua allowed by string theory the unique ground state that corresponds to the known coupling constants of the universe. All free parameters are fixed in the sense that if the fundamental parameters (masses, coupling constants, and mixing angles) had different values, larger vacuum energy would arise.

9 Conclusion

We have presented a new model for dark energy in the universe. This model is based on a rather conservative approach, the assumption of the existence of second quantized self-interacting scalar fields described by a φ^4-theory. However, the main difference is that these fields are strongly self-interacting, rather than weakly. When doing 2nd quantization using the Parisi-Wu approach, rapidly fluctuating chaotic fields arise. The expectation of the underlying potentials yields the currently observed dark energy density.

The advantage of this new chaotic model is that many of the questions raised in the introduction seem to have natural solutions. The cosmological constant problem is avoided, in our model the right order of magnitude of vacuum energy is naturally produced if we interpret the chaotic dynamics in terms of vacuum fluctuations allowed by the uncertainty relation, for a given finite age of the universe. The cosmological coincidence problem is also avoided, since in our model the parameters of the scalar potentials of the fields are explicitly time-dependent, and hence the generated vacuum energy is not constant anymore, but thins out with the expansion of the universe in the same way as the energy density of the dominating species (matter or radiation). In spite of that, the (classical) equation of state of the chaotic component is still close to $w = -1$, and can account for the accelerated expansion of the universe. The chaotic fields are associated with vacuum fluctuations and hence violate energy conservation.

The physical relevance of our model is emphasized by the observation of a large number of numerical coincidences between local minima in the dark energy landscape and running standard model coupling constants evaluated at the known fermionic and bosonic mass scales. It thus appears that chaotic fields have the potential to fix and stabilize fundamental parameters and to select the physically relevant vacuum state out of infinitely many possibilities.

Appendix A: General Tchebyscheff maps

Our approach can be easily generalized to Tchebyscheff maps of arbitrary order N. One has $T_1(\Phi) = \Phi$, $T_2(\Phi) = 2\Phi^2 - 1$, $T_3(\Phi) = 4\Phi^3 - 3\Phi$, generally $T_N(\Phi) = \cos(N \arccos \Phi)$ with $\Phi \in [-1, 1]$. A Tchebyscheff map of order N is conjugated to a Bernoulli shift of N symbols, it is ergodic and mixing for $N \geq 2$. It exhibits the strongest possible chaotic behaviour that is possible for a 1-d smooth map, characterized by a minimum sceleton of higher-order correlations [20].

It is useful to consider both positive and negative Tchebyscheff maps and to define

$$T_{-N}(\Phi) := -T_N(\Phi). \tag{71}$$

The behaviour of T_{-N} under iteration is identical to that of T_N up to a sign, the trajectory of T_{-N} differs by a constant sign (N even) or an alternating sign (N odd) from that of T_N.

Eq. (13) implies that the maps T_N correspond to potentials V_N given by

$$V_N(\varphi) = \frac{1-\alpha}{\tau}\left\{-\frac{1}{2}\varphi^2 + p_{max}\int d\varphi\, T_N\left(\frac{\varphi}{p_{max}}\right)\right\} + const \tag{72}$$

In particular, one obtains for $N = \pm 1, \pm 2, \pm 3$

$$\begin{aligned}
V_{\pm 1}(\varphi) &= \frac{1-\alpha}{\tau}\left(-\frac{1}{2}\varphi^2 - (\pm\frac{1}{2}\varphi^2)\right) + const && (73)\\
V_{\pm 2}(\varphi) &= \frac{1-\alpha}{\tau}\left(-\frac{1}{2}\varphi^2 \pm (\frac{2}{3p_{max}}\varphi^3 - p_{max}\varphi)\right) + const && (74)\\
V_{\pm 3}(\varphi) &= \frac{1-\alpha}{\tau}\left(-\frac{1}{2}\varphi^2 - \pm(-\frac{3}{2}\varphi^2 + \frac{1}{p_{max}^2}\varphi^4)\right) + const. && (75)
\end{aligned}$$

Of physical relevance are the expectations of these potentials, formed with respect to the ergodic dynamics. Since negative and positive Tchebyscheff maps generate essentially the same dynamics, up to a sign, any physically relevant expecatation should also be the same for T_N and T_{-N}, up to a possible sign. For all N, this symmetry condition fixes the additive constant to be

$$const = +\frac{1-\alpha}{\tau}\frac{1}{2}\langle\varphi^2\rangle \tag{76}$$

With this choice one obtains the following formulas for the self energy which are fully symmetric under the transformation $N \to -N$:

$$\begin{aligned}
\langle V_{\pm 1}(\varphi)\rangle &= \pm\frac{1-\alpha}{\tau}\frac{1}{2}\langle\varphi^2\rangle && (77)\\
\langle V_{\pm 2}(\varphi)\rangle &= \frac{1-\alpha}{\tau}\left(\frac{2}{3p_{max}}\langle\varphi^3\rangle - p_{max}\langle\varphi\rangle\right) && (78)\\
\langle V_{\pm 3}(\varphi)\rangle &= \pm\frac{1-\alpha}{\tau}\left(-\frac{3}{2}\langle\varphi^2\rangle + \frac{1}{p_{max}^2}\langle\varphi^4\rangle\right) && (79)
\end{aligned}$$

Written in terms of the dimensionless field variable $\Phi = \varphi/p_{max}$ this is

$$\begin{aligned}
\langle V_{\pm 1}(\varphi)\rangle &= \pm\frac{1-\alpha}{\tau}p_{max}^2\frac{1}{2}\langle\Phi^2\rangle && (80)\\
\langle V_{\pm 2}(\varphi)\rangle &= \frac{1-\alpha}{\tau}p_{max}^2\left(\frac{2}{3}\langle\Phi^3\rangle - \langle\Phi\rangle\right) && (81)\\
\langle V_{\pm 3}(\varphi)\rangle &= \pm\frac{1-\alpha}{\tau}p_{max}^2\left(-\frac{3}{2}\langle\Phi^2\rangle + \langle\Phi^4\rangle\right). && (82)
\end{aligned}$$

For Tchebyscheff maps of arbitrary order N one obtains

$$\begin{aligned}
V_{\pm N}(\varphi) &= \frac{1-\alpha}{\tau}p_{max}^2\left\{-\frac{1}{2}\Phi^2 \pm \int \cos(N\arccos\Phi)d\Phi\right\} && (83)\\
&= \frac{1-\alpha}{2\tau}p_{max}^2\left\{-\Phi^2 \pm \left(\frac{1}{N+1}T_{N+1}(\Phi) - \frac{1}{N-1}T_{N-1}(\Phi)\right)\right\}\\
&\quad +const && (84)
\end{aligned}$$

and

$$\langle V_{\pm N}(\varphi)\rangle = (\pm 1)^N\frac{1-a}{2\tau}p_{max}^2\left\{\frac{1}{N+1}\langle T_{N+1}(\Phi)\rangle - \frac{1}{N-1}\langle T_{N-1}(\Phi)\rangle + C\right\}. \quad (85)$$

For uncoupled Tchebyscheff maps with $|N| \geq 2$, any expectation of an observable $A(\Phi)$ is given by eq. (19) and (20). For $\alpha \neq 0$ the invariant density changes in a nontrivial way, but expectations can still be easily calculated numerically by long-time iteration of the coupled map lattice.

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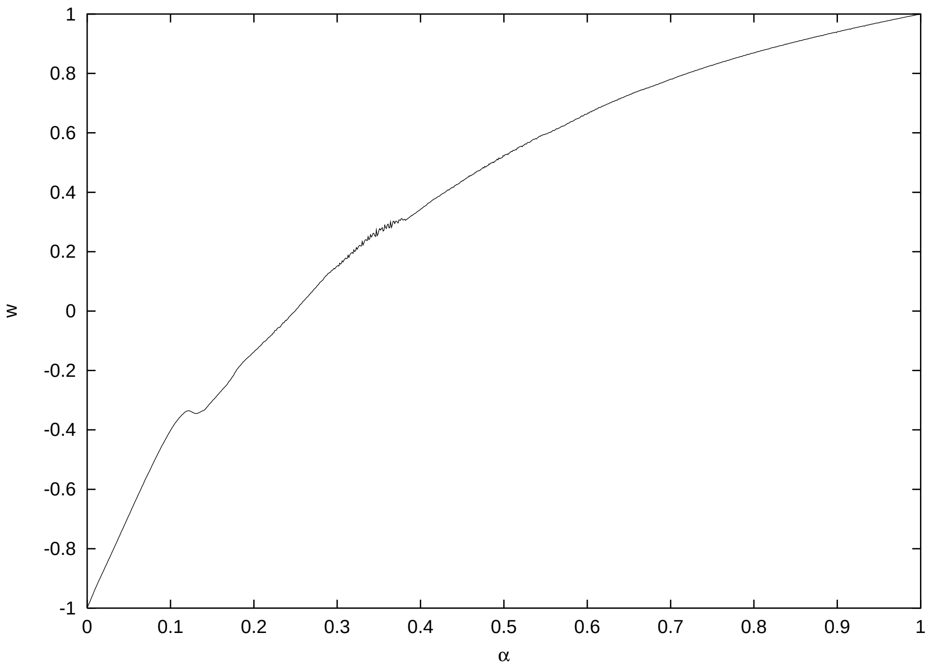

Fig. 1 Classical equation of state $w = \langle p \rangle / \langle \rho \rangle$ of the chaotic field φ as a function of the coupling α.

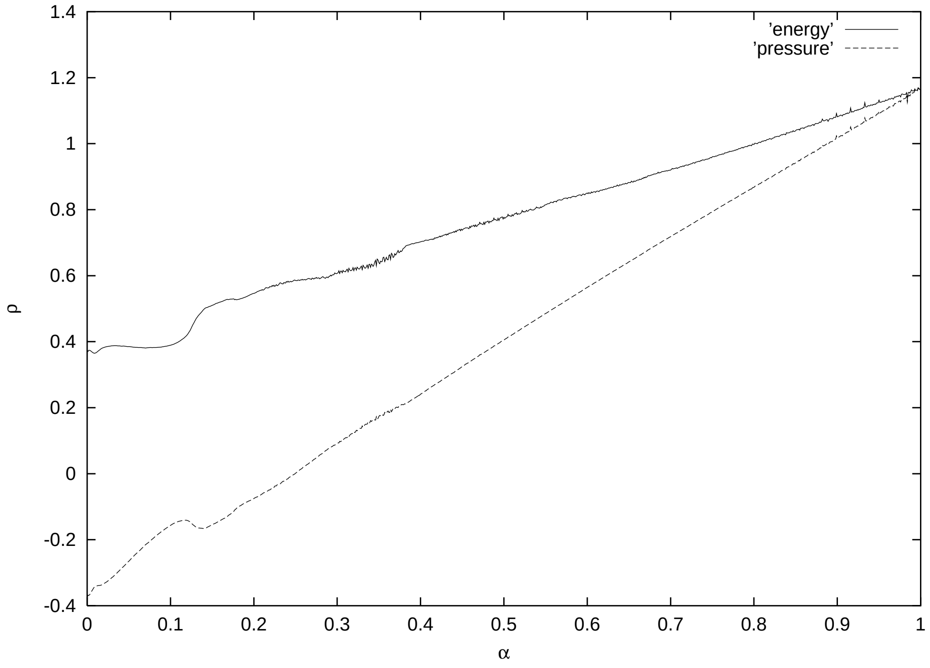

Fig. 2 Expectation of energy and pressure of the chaotic field as a function of the coupling α.

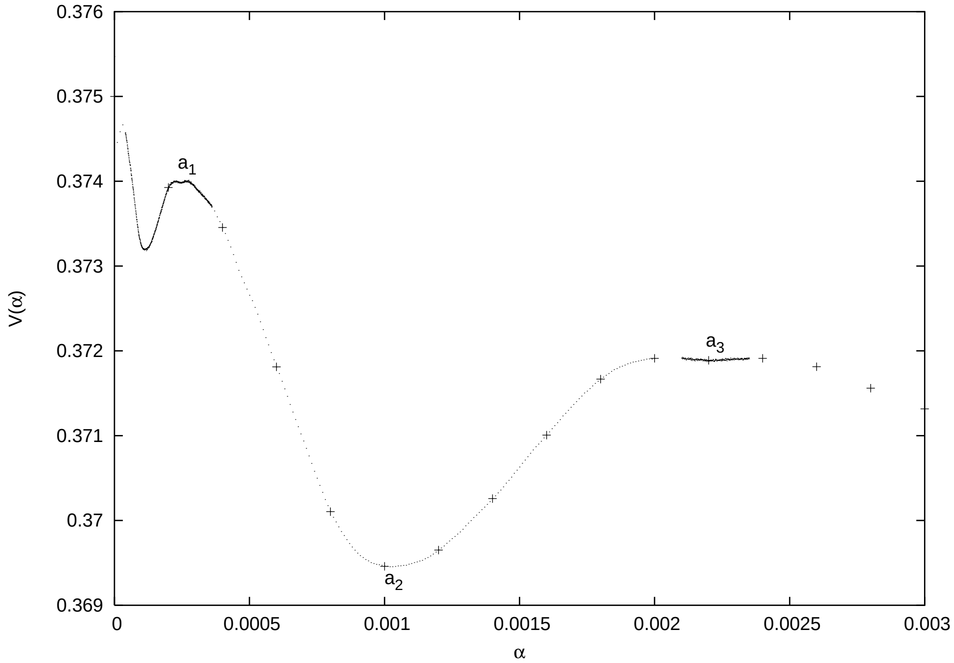

Fig. 3 Self energy $V(\alpha)$ of the type-A chaotic field in the low-coupling region. There are local minima at couplings a_i that coincide with the weak coupling constants of right-handed fermions in the standard model.