THE SINGULARITY SPECTRUM OF THE FISH'S BOUNDARY

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Let $M(\mathbb{T}^1, T)$ be the convex set of Borel probability measures on the Circle \mathbb{T}^1 invariant by the doubling map T. Its projection on the complex plane by the application $\mu \mapsto \int e^{2i\pi x} d\mu(x)$ is a compact convex subset of the unit disc, symmetric with respect to the horizontal axis, called the "Fish" by T. Bousch. Seeing the boundary of the upper half-Fish as a function, we focus on its local regularity. We show that the multifractal spectrum is concentrated at infinity, but that every pointwise regularity $\alpha \in [1, \infty]$ is realized in a non-denumerable and dense set of points. The results rely on fine properties of Sturm measures.