Counting colourings

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The chromatic polynomial $P_X(q)$ of a graph X has the property that its value at a positive integer q is the number of proper colourings of the graph with q colours. However, the number it gives you may not be the number you want, for two reasons:

- it doesn't allow for the fact that different colourings may be equivalent under an automorphism of the graph;
- the order of the colours is significant, whereas you may want to count the partitions of the graph into independent sets.

I will present separate solutions for these two problems; one uses the *orbital chromatic polynomial*, while the other is a simple inclusion-exclusion argument. However, combining the two methods to count partitions into independent sets up to automorphisms is an unsolved problem.

Another feature of the chromatic polynomial is that substituting -1 for q counts the number of acyclic orientations of the graph. It turns out that substituting -1 for q in the orbital chromatic polynomial does not count orbits on acyclic orientations; but there is a "twisted" version of the orbital chromatic polynomial which does this job.

These things will be illustrated by means of the Petersen graph.