Totally frustrated states: A physics-like generalisation of graph colouring

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When counting the proper colourisations of a graph, whose number is given by the chromatic polynomial, all one needs to know is the number of colours in the colour set; the nature of the individual colours is immaterial. When colouring a *gain graph*, where the edges are labelled by elements of a group (called the *gain group*), that is no longer so. In order for the concept of a proper colourisation to be meaningful the group must have a permutation action on the set of colours, and then the exact way the group acts is a crucial factor in counting proper colourations.

One reason to study gain graph colouring is that it generalises simple physical models of spin glasses like the Ising model, in which the edges are labelled from the sign group and there are two colours, and the Potts model, in which there are more colours. Due to this connection we call the colours 'spins' and the generalised proper colourisation a *totally frustrated state*.

The number of totally frustrated states is not related to a matroid invariant (as far as I know), except in ordinary graph colouring and a previously known gain-graphic generalisation, but it always satisfies a deletion-contraction law and it has a formula in terms of holonomy groups of edge subsets.