A Web-based Resource for Design Theory

1 Description of the project

We believe it is important and timely to create a comprensive world-wide-web resource for Design Theory, covering combinatorial, computational and statistical aspects of design. We also propose to develop algorithms and software (in the form of a GAP share-package) for generating, classifying and studying combinatorial designs. We plan to use this software to publish a database of designs on the world-wide-web, as well as publishing theoretical advances in more conventional print format. The total duration for this research project will be three years.

Our web-resource will provide:

- an on-line database of (downloadable) combinatorial designs, useful for mathematicians and statisticians in academia and industry,
- online access to software for generating and analyzing designs,
- new designs and constructions arising from research into design theory and related topics at Queen Mary, University of London,
- comprehensive information on combinatorial, computational and statistical aspects of design theory, the contents of which we plan to make the basis of an accompanying book,
- comprehensive links to design theory resources at Queen Mary and throughout the world.

We remark that The CRC Handbook of Combinatorial Designs [2] is a very valuable resource for combinatorial design theory and we will make good use of it. However, it is a tersely presented print-only reference book.

2 Staff

Dr L.H. Soicher will spend approximately fifteen hours per week on the project. He will be responsible for project management and will provide expertise in discrete mathematical computation in general and programming in GAP in particular.

Prof. P.J. Cameron will spend approximately ten hours per week on the project. He will will provide expertise in many aspects of combinatorics and algebra.

Prof. R.A. Bailey will spend approximately five hours per week on the project. She will provide expertise in many aspects of statistics and design of experiments.

We are also asking for funding for a Postdoctoral Research Assistant and a PhD studentship for the three-year duration of the project.

The task of the PDRA will depend on his/her areas of expertise (which we expect to be combinatorics, statistics or mathematical computation), but will include development of the design theory web-site, design and programming of the GAP share package for designs, and research into design theory. The PDRA will need to learn new disciplines as appropriate and interactions between these disciplines (combinatorial design theory, statistics, group theory, algorithm design and implementation, database design, web-site design).

The PhD student will be given a task suited to his/her background and interests, and could include algorithm development and implementation, and research into the construction of families of new designs.

3 Interdisciplinary interactions

Our web-resource will feature the integration of the theoretical and practical approaches to design theory, and the integration of the theoretical and the computational approaches to the construction and investigation of particular designs. We have special expertise in both these interfaces.

In the last decade, computer algebra systems such as GAP [3] and MAGMA [1], originally designed for investigating algebraic structures such as groups, have broadened their areas of application into designs, graphs, and other parts of discrete mathematics. We shall be furthering these advances.

We shall also be seeking input from Computer Scientists on the topics of internet-database design, human-computer interaction, and randomized and heuristic algorithms for hard search problems (for finding designs). Indeed, the Queen Mary Computer Science Department has a research group in Information, Media and Communication, and our Electronic Engineering Department offers an MSc in Internet Computing, so some of this expertise is right at home.

4 GAP, share packages, GRAPE, and DESIGN

GAP is an extensive, powerful, user-programmable system for computational discrete algebra (with particular emphasis on, but not restricted to, computational group theory). The development of GAP is an international enterprise, which is coordinated and maintained by the School of Mathematical and Computational Sciences at the University of St. Andrews. GAP and its sources, including share packages, data library and the manual, are distributed freely for non-commercial purposes.

GAP share packages are user contributed modules covering specific areas of discrete algebra. Some written in entirely in the GAP language; others include and provide interfaces to external programs. All share-package functions can be called directly from GAP as if they were library functions, and their documentation is provided on-line in GAP, as for library functions.

A "GAP-Council" of experts, of which L.H. Soicher is a member, has been established which not only advises the further development of GAP, but also acts as an editorial board for share packages, which are formally refereed.

One of the very first and most heavily used share packages is GRAPE [5], written and developed by L.H. Soicher. GRAPE computes with graphs together with groups of automorphisms of those graphs. This gives (often very dramatic) savings in store and CPU-time, but also, importantly, helps a mathematician work with graphs from a group-theoretical or geometrical perspective. GRAPE has been used successfully in much research, notably the study of distance-regular graphs. More recently, GRAPE has been applied to the discovery and classification of new combinatorial designs [6].

There is a demand for a GAP share package for designs, along the lines of GRAPE. We propose to provide such a package, called DESIGN, which will provide standard constructions for combinatorial designs, functions to study and classify designs, and an interface to the *nauty* package [4] (as in GRAPE) to compute automorphism groups of designs. We plan also to provide interfaces to statistical packages such as GENSTAT, for the statistical analysis of designs, and to make the DESIGN package available interactively through our web-site.

5 Queen Mary design theory online

We already have a little experience in web-publishing research on designs, providing designs online, and presenting web-reources for designs.

Our preliminary page for web-resources for designs can be found at
http://www.maths.qmw.ac.uk/~pjc/design/resources.html
For some information and research on certain types of designs see
http://www.maths.qmw.ac.uk/~rab/sls.html
and http://www.maths.qmw.ac.uk/~leonard/soma/

Our planned web-site will expand greatly on these very preliminary resources, providing a uniform, comprehensive, attractive, interactive web-resource for design theory.

References

[1] W. Bosma, J. Cannon and C. Playoust, The MAGMA algebra system I: The user language, *J. Symbol. Comput.*, **24** (1997), 235–265.

[2] C.J. Colbourn and J. Dinitz (editors), The CRC Handbook of Combinatorial Designs, CRC Press, Boca Raton, 1996.

[3] The GAP Group, Lehrstuhl D für Mathematik, RWTH Aachen, Germany and School of Mathematical and Computational Sciences, U. St. Andrews, Scotland, *GAP – Groups, Algorithms, and Programming, Version 4*, 1999. GAP is available from
http://www-gap.dcs.st-and.ac.uk/~gap/.

[4] B.D. McKay, *nauty* user's guide (version 1.5), Technical report TR-CS-90-02, Computer Science Department, Australian National University, 1990. *nauty* is available within GRAPE or from
http://cs.anu.edu.au/~bdm/nauty/.

[5] L.H. Soicher, GRAPE: a system for computing with graphs and groups, in *Groups and Computation* (L. Finkelstein and W.M. Kantor, eds), DIMACS Series in Discrete Mathematics and Theoretical Computer Science **11**, AMS, 1993, pp. 287–291. GRAPE is available from
http://www-gap.dcs.st-and.ac.uk/~gap/Share/grape.html.

[6] L.H. Soicher, On the structure and classification of SOMAs: generalizations of mutually orthogonal Latin squares, *Electronic J. Combinatorics* **6** (1999), R32, 15 pp. available from
http://www.combinatorics.org