Review of *Before Sudoku: The World of Magic Squares* by Seymour S. Block and Santiago A. Tavares, Oxford University Press, 2009.

A few years ago, a supplement on Switzerland in a national newspaper included, in an item "Ten things you didn't know about Switzerland", the information that Euler invented Sudoku. In fact he didn't, but he took a crucial step on the road from magic squares (which have fascinated humankind for thousands of years) to Sudoku.

Euler wrote two papers on Latin squares. A Latin square is a square array of size $n \times n$ with entries from an alphabet of size n, with the property that each alphabet symbol occurs once in each row and once in each column (think Sudoku without the constraint on subsquares.) A Graeco-Latin square consists of a pair of Latin squares of the same size, such that each pair of symbols (one from the alphabet of each square) occurs in precisely one position. Euler envisaged using the Latin and Greek alphabets for the two squares.

He constructed Graeco-Latin squares of all orders not congruent to 2 mod 4. He also remarked that, given a Graeco-Latin square of order n, if we take both alphabets to be $\{0, 1, \ldots, n-1\}$ (the digits in base n), and interpret each entry as a two-digit integer in base n, and then add 1 to each entry, then we have filled the cells of the square with the integers $1, 2, \ldots, n^2$ in such a way that all row and column sums are $n(n^2+1)/2$. With a bit of Eulerian cleverness, we can usually ensure that each diagonal also has this sum.

Such an object is a *magic square*. Euler's motivation was to add a new construction to the very substantial literature on magic squares. These had been considered by Chinese, Indian and Arab mathematicians, and many different constructions were already known. Part of their importance came from their use as talismans, which also may have given rise to the name.

In a second paper, Euler considered Latin squares in their own right. Since then, Latin squares have found applications in algebra, statistics, and coding theory, and have become mainstream mathematics, while magic squares are mostly the preserve of recreational mathematicians.

To conclude the story of Sudoku: In 1956 the statistician W. Behrens defined *gerechte designs*, or Latin squares whose cells are partitioned into n regions, each symbol occurring once in each region; and in 1977 the statistician John Nelder defined *critical sets* in Latin squares, sets of cells minimal with respect to being completable in a unique way. No statistician thought to combine these ideas, and it was a retired American architect, Howard Garns, who invented the puzzle Number Place in 1987. The puzzle was introduced to Japan by Maki Kaji, and re-introduced to the West by New Zealander Wayne Gould (brother of former British politician Brian Gould). At this point it became a world-wide craze.

The book under review traces some of this history (omitting the contributions of Behrens and Nelder), but then turns to magic squares and their variants. The variants are of several types:

- We may impose stronger conditions: for example, in a 4×4 square, we might also require the entries in the 2×2 subsquares to sum to the magic constant 34.
- We may replace the rows, columns and diagonals of a square by other sets, usually defined geometrically by lines or circles in other geometric figures.
- Instead of, or as well as, constant sums, we may ask for constant products, or sums of squares or higher powers.
- Some magic squares can be transformed into others by reflection or other manipulation.

Many examples are given, but the book gives little insight into how the constructions are done. The authors' attitude is typefied by a statement quoted approvingly from the architect Claude Bragdon:

> Ours is the age of mathematics, it is the magician's wand without which our workers of magic, be they bankers, engineers, physicists, inventors could not perform their tricks.

Uses of magic squares in the arts (including the music of Peter Maxwell Davies and others) and the sciences (experimental design and error-correction – these really use Latin squares) are described.

The book is marred by several careless errors:

- In a single clause, we are told that the number of ways of writing the numbers $1, 2, \ldots, 64$ in an 8×8 array is more than 10^{88} (a simple calculation), and that this is the number of miles to the edge of the universe (which is certainly wrong); the whole is attributed to Martin Gardner.

- On page 50, a magic hexagram involving fractions, taken from a grade five exercise booklet, is given. The authors do all the sums, but get one wrong.

- The magic square in Dürer's engraving *Melencolia I*, illustrated on page 26, is attributed to Benjamin Franklin on page 79; later it becomes the Dürer-Franklin square.

- R. A. Fisher was at Rothamsted Experimental Station, not the University of Cambridge, when he pioneered the use of Latin squares in agricultural experiments. In fact he was not the first to do so: Donald Preece informs me that Cretté de Palluel published a study on feeding roots to sheep in 1788.

- The name of the puzzle masters Henry Ernest Dudeney and Sam Loyd are mis-spelt throughout the book.

It seems odd that "Olympic records" for the largest magic squares constructed by hand are given, when Arab mathematicians knew several general constructions. (But Indian and Arab mathematicians have been written out of the story here.) It is also odd that the work of Kathleen Ollerenshaw and her collaborator David Brée on the classification of magic squares is not mentioned.

The authors explain that the book is for people who enjoy Sudoku puzzles. Sad to say, it is not for mathematicians.

Peter J. Cameron