Let f be a function on the natural numbers with the properties

- *n divides $f(n)$;*
- *there exists a function g such that $g(m)$ divides n if and only if m divides $f(n)$.*

If (u_n) is realizable, then so is $(u_{f(n)})$.

For let (X,T) be a realization of (u_n). Let $X^* = X$, and construct T^* as follows: replace each cycle of T (of length n, say) by its $n/g(n)$th power (which has $n/g(n)$ cycles of length $g(n)$). (Note that $g(n)$ divides n, by assumption.) Now let $x \in X$ be a point lying in an m-cycle of T. Then x lies in a $g(m)$-cycle of T^*, so

$$\begin{aligned} (T^*)^n \text{ fixes } x &\Leftrightarrow g(m) \text{ divides } n \\ &\Leftrightarrow m \text{ divides } f(n) \\ &\Leftrightarrow T^{f(n)} \text{ fixes } x. \end{aligned}$$

Now let $f(n) = n^k$. Define g to be the multiplicative function satisfying

$$g(p^a) = p^{\lceil a/k \rceil}$$

for p prime and $a \geq 0$. Suppose that $n = p_1^{a_1} \cdots p_r^{a_r}$ and $m = p_1^{b_1} \cdots p_r^{b_r}$. Then

$$m \text{ divides } n^k \Leftrightarrow (\forall i) b_i \leq k a_i,$$

and

$$n \text{ divides } g(m) \Leftrightarrow (\forall i) \lceil b_i/k \rceil \leq a_i;$$

these conditions are clearly equivalent.