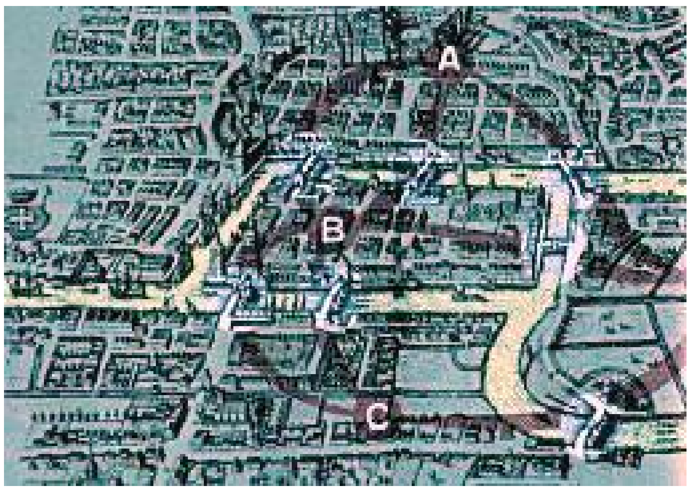
Discrete Mathematics



AAA BBB CCC … *NNN OOO PPP*
HGB GHA FED … *KLM JIP IJO*
KJD LIC ILB … *HEO EHN FGM*
MIE NJF OKG … *BFJ CGK DHL*
OFL PEK MHJ … *DIG ALF BKE*
PCN ODM NAP … *CPA BMD ANC*
BDC ACD DBA … *MOP PNM OMN*
CKI DLJ AIK … *PHF MEG NFH*
DMP CNO BON … *OBC NCB MDA*
EBF FAE GDH … *JMI KPL LOK*
FLO EKP HJM … *IGD LFA KEB*
GPJ HOI ENL … *LCE IBH JAG*
IEM JFN KGO … *FJB GKC HLD*
JOH IPG LMF KNE NKD MLC PIB OJA BGP AHO DEN CFM FCL EDK HAJ GBI
LNG KMH JPE IOF PJC OID NLA MKD DFO CEP BHM AGN HBK GAL FDI ECJ
NHK MGL PFI OEJ JDO ICP LBM KAN FPC EOD HNA GMB BLG AKH DJE CIF

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What’s in a name?

From the Edexcel website:

Unit D1: Decision Mathematics

1. Algorithms; 2. Algorithms on graphs; 3. The route inspection problem; 4. Critical path analysis; 5. Linear programming; 6. Matchings; 7. Flows in networks

Unit D2: Decision Mathematics

1. Transportation problems; 2. Allocation (assignment) problems; 3. The travelling salesman problem; 4. Game theory; 5. Dynamic programming

The term “Finite Mathematics” is also used for similar material (mainly in the USA).

Think of a number . . .

Think of a number between 0 and 15.

Now answer the following questions.

You are allowed to lie **once**.

1. Is the number 8 or greater?
2. Is it in the set $\{4, 5, 6, 7, 12, 13, 14, 15\}$?
3. Is it in the set $\{2, 3, 6, 7, 10, 11, 14, 15\}$?
4. Is it odd?
5. Is it in the set $\{1, 2, 4, 7, 9, 10, 12, 15\}$?
6. Is it in the set $\{1, 2, 5, 6, 8, 11, 12, 15\}$?
7. Is it in the set $\{1, 3, 4, 6, 8, 10, 13, 15\}$?

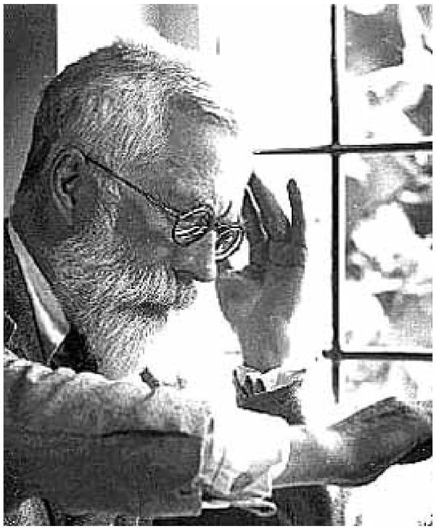
Why the trick works

The possible answers to the seven questions, if no lies are told, are:

0	0	0	0	0	0	0	0
1	0	0	0	1	1	1	1
2	0	0	1	0	1	1	0
3	0	0	1	1	0	0	1
4	0	1	0	0	1	0	1
5	0	1	0	1	0	1	0
6	0	1	1	0	0	1	1
7	0	1	1	1	1	0	0
8	1	0	0	0	0	1	1
9	1	0	0	1	1	0	0
10	1	0	1	0	1	0	1
11	1	0	1	1	0	1	0
12	1	1	0	0	1	1	0
13	1	1	0	1	0	0	1
14	1	1	1	0	0	0	0
15	1	1	1	1	1	1	1

You can see that any two rows differ in at least three places. So if one lie is told, the responses are "closer" to the correct answer than to any other.

Not just a game

To the statistician R. A. Fisher, it is a *fractional factorial design*; to the electrical engineer R. W. Hamming, it is an *error-correcting code*.

To an algebraist, it is a subspace of the 7-dimensional vector space over the *binary field* $\mathbb{F}_2 = \{0, 1\}$.

To a geometer, there is a close relationship with projective geometry.

Continuous or discrete?

It may not be a coincidence that the two systems in the universe that most impress us with their open-ended complex design – life and mind – are based on discrete combinatorial systems.

> Steven Pinker, *The Language Instinct*, Penguin, London, 1994.

The emphasis on mathematical methods seems to be shifted more towards combinatorics and set theory – and away from the algorithm of differential equations which dominates mathematical physics.

> J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton University Press, Princeton, 1944.

He [Maynard Smith] saw the need for "some kind of concept that in dynamical systems there are going to be sudden breaks and thresholds and transformations, and so on". He added that, in his opinion, "today we really do have a mathematics for thinking about complex systems and things which undergo transformations from quantity into quality". Here he saw Hopf bifurcations and catastrophe theory as really nothing other than a change of quantity into quality in a dialectical sense.

> Ullica Segerstråle, *Defenders of the Truth: The Battle for Science in the Sociobiology Debate and Beyond*, Oxford University Press, Oxford, 2000.

MAS105 Discrete Mathematics

Description

A collection of topics giving a flavour of some of the algebraic and geometric structures covered in later courses.

Induction; arithmetic, geometric and binomial series.

Sets and functions.

Number theory: divisibility, prime factorisation, congruences, fields GF(p).

Polynomials: division theorem, roots, irreducibility, factorisation, construction of prime power fields.

Graph theory: paths, cycles, trees, Euler's theorem.

Permutations: orbits, composition, order, inverses, parity, odd-even theorem.

Plane isometries: types, symmetries, classification by reflections.

Career Related Article

Should You Prepare Differently for a Non-academic Career?

Fan R. K. Chung, Bellcore

. . .

In the past twenty years, we have been in the midst of a technological revolution. . . . To deal with problems of astronomical size and complexity, clever methods and powerful tools are in great need. . . . Mathematics will have significant impact over the entire spectrum of developments in the next round of the technological revolution, from establishing information infrastructures to software research.

What should a student do to get ready for the interesting and exciting period of mathematics lying ahead of us? In addition to learning as much mathematics as you can, there are areas which deserve special attention. Discrete mathematics, the study of fundamental properties of discrete structures, has now evolved into a rich and dynamic discipline with growing connections to other areas of mathematics and computer science. It would be advantageous for a graduate student (including those majoring in areas of continuous mathematics) to have some exposure to discrete mathematics such as combinatorics, graph theory and number theory, preferably beyond the introductory level. In addition, there are several other courses worth recommending such as geometry with a computational flavor (or with imagination), probability (or combinatorial probabilistic methods), numerical methods or even interdisciplinary courses in algorithms and data structures. However, it is sad to note that very few universities offer many of the courses mentioned above.