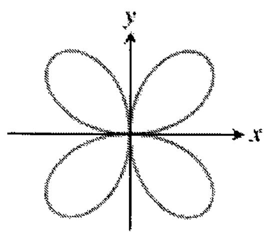
MAS115 Calculus I 2006-2007

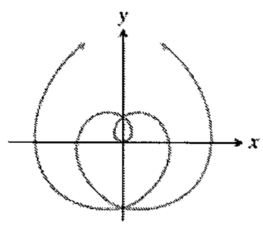
Problem sheet for exercise class 9

Four-leaved rose



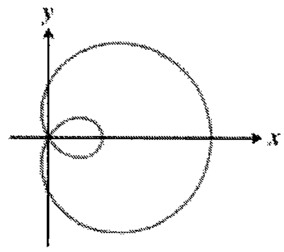
(*)

Spiral



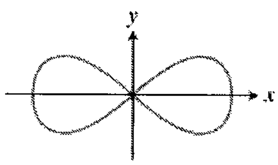
(*)

Limaçon

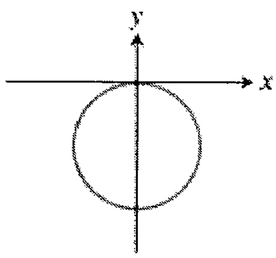


(*)

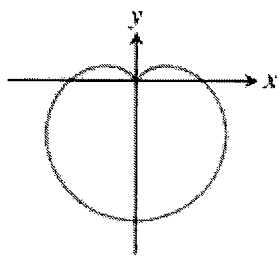
Lemniscate



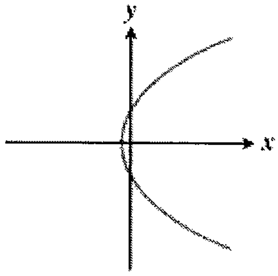
Circle



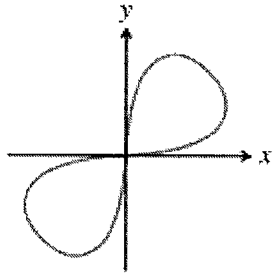
Cardioid



Parabola



Lemniscate



Problem 1: Match each of the eight graphs with one of the following equations.

a.	$r = \cos 2\theta$,	**b.**	$r \cos\theta = 1$,	**c.**	$r = \dfrac{6}{1 - 2\cos\theta}$,
d.	$r = \sin 2\theta$,	**e.**	$r = \theta$,	**f.**	$r^2 = \cos 2\theta$,
g.	$r = 1 + \cos\theta$,	**h.**	$r = 1 - \sin\theta$,	**i.**	$r = \dfrac{2}{1 - \cos\theta}$,
j.	$r^2 = \sin 2\theta$,	**k.**	$r = -\sin\theta$,	**l.**	$r = 2\cos\theta + 1$.

Problem 2: Show that the equations $x = r\cos\theta$, $y = r\sin\theta$ transform the polar equation

$$r = \frac{k}{1 + e\cos\theta}$$

into the Cartesian equation

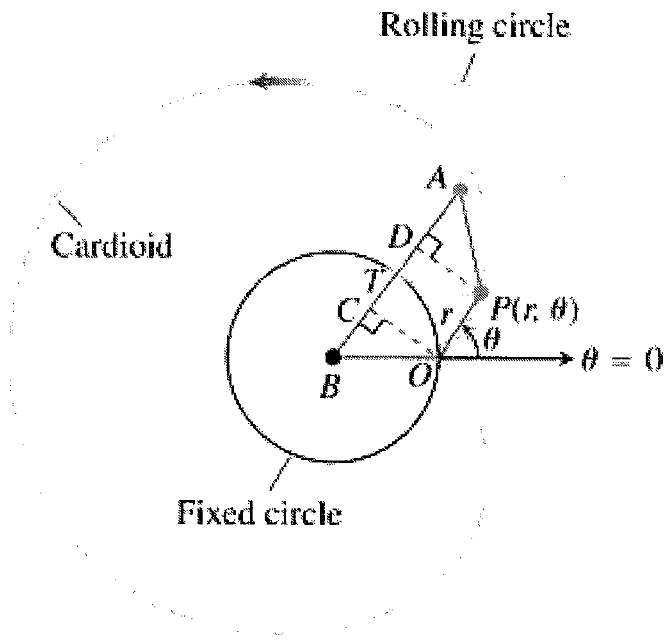
$$(1 - e^2)x^2 + y^2 + 2kex - k^2 = 0\ .$$

Problem 3: Find polar equations for the following four circles. Sketch each circle in the coordinate plane and label it with both its Cartesian and polar equations.

a. $x^2 + y^2 + 5y = 0\ ,$ **b.** $x^2 + y^2 - 2y = 0\ ,$

c. $x^2 + y^2 - 3x = 0\ ,$ **d.** $x^2 + y^2 + 4x = 0\ .$

Extra: Show that if you roll a circle of radius a about another circle of radius a in the polar coordinate plane, the original point of contact P will trace a cardioid. (Hint: start by showing that $\angle OBC$ and $\angle PAD$ are equal to each other.)



Thomas Prellberg, December 2006

Problem 1

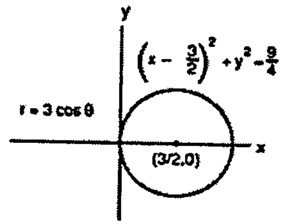
(*) rose : d	(*) spiral : e	(*) limaçon : l
lemniscate : f	circle : k	cardioid : h
parabola : i		lemniscate (diagonal) : j

Problem 2

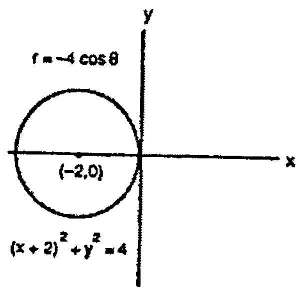
$$r = \frac{k}{1 + e\cos\theta} \Rightarrow r + er\cos\theta = k \Rightarrow \sqrt{x^2 + y^2} + ex = k \Rightarrow \sqrt{x^2 + y^2} = k - ex \Rightarrow x^2 + y^2$$
$$= k^2 - 2kex + e^2x^2 \Rightarrow x^2 - e^2x^2 + y^2 + 2kex - k^2 = 0 \Rightarrow (1 - e^2)\,x^2 + y^2 + 2kex - k^2 = 0$$

Problem 3

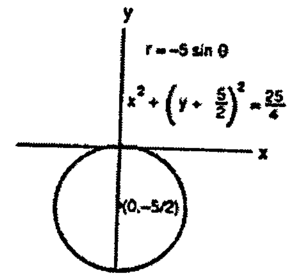
$x^2 + y^2 - 3x = 0 \Rightarrow \left(x - \frac{3}{2}\right)^2 + y^2 = \frac{9}{4} \Rightarrow C = \left(\frac{3}{2}, 0\right)$
and $a = \frac{3}{2}$: $r^2 - 3r\cos\theta = 0 \Rightarrow r = 3\cos\theta$



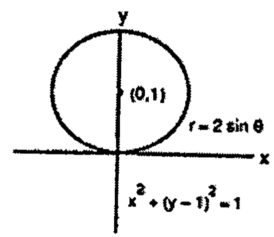
$x^2 + y^2 + 4x = 0 \Rightarrow (x + 2)^2 + y^2 = 4 \Rightarrow C = (-2, 0)$
and $a = 2$: $r^2 + 4r\cos\theta = 0 \Rightarrow r = -4\cos\theta$



$x^2 + y^2 + 5y = 0 \Rightarrow x^2 + \left(y + \frac{5}{2}\right)^2 = \frac{25}{4} \Rightarrow C = \left(0, -\frac{5}{2}\right)$
and $a = \frac{5}{2}$: $r^2 + 5r\sin\theta = 0 \Rightarrow r = -5\sin\theta$



$x^2 + y^2 - 2y = 0 \Rightarrow x^2 + (y - 1)^2 = 1 \Rightarrow C = (0, 1)$ and
$a = 1$: $r^2 - 2r\sin\theta = 0 \Rightarrow r = 2\sin\theta$



Extra:

Arc PT = Arc TO since each is the same distance rolled. Now Arc PT $= a(\angle TAP)$ and Arc TO $= a(\angle TBO)$ $\Rightarrow \angle TAP = \angle TBO$. Since $AP = a = BO$ we have that ΔADP is congruent to $\Delta BCO \Rightarrow CO = DP \Rightarrow OP$ is parallel to $AB \Rightarrow \angle TBO = \angle TAP = \theta$. Then OPDC is a square $\Rightarrow r = CD = AB - AD - CB = AB - 2CB$ $\Rightarrow r = 2a - 2a\cos\theta = 2a(1 - \cos\theta)$, which is the polar equation of a cardioid.