MAS115 Calculus I 2006-2007

Problem sheet for exercise class 9

- **Make sure you attend the excercise class that you have been assigned to!**
- Try to work on the problems first on your own. If you are stuck, ask for hints.
- The instructor and helper will be available for questions.
- Solutions will be available online by Friday.

Problem 1: **Making a simplifying substitution.** Evaluate

$$\int_0^{\sqrt{\ln 2}} 2xe^{x^2}\,dx\ .$$

Problem 2: **Completing the square.** Evaluate

$$\int \frac{d\theta}{\sqrt{2\theta-\theta^2}}\ .$$

Problem 3: **Using a trigonometric identity.** Evaluate

$$\int (\sin 3x \cos 2x - \cos 3x \sin 2x)dx\ .$$

Problem 4: **Eliminating a square root.** Evaluate

$$\int_{-\pi}^{0} \sqrt{1-\cos^2\theta}\,d\theta\ .$$

Problem 5: **Reducing an improper fraction.** Evaluate

$$\int_{\sqrt{2}}^{3} \frac{2x^3}{x^2-1}dx\ .$$

Problem 6: **Separating a fraction.** Evaluate

$$\int \frac{1-x}{\sqrt{1-x^2}}dx\ .$$

Problem 7: **Multiplying by 1.** Evaluate

$$\int \frac{1}{1+\sin x}dx\ .$$

Extra: For what $x>0$ does $x^{x^x}=(x^x)^x$ hold?

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Problem 1

$$\int_0^{\sqrt{\ln 2}} 2xe^{x^2}\,dx;\ \left[\begin{array}{c} u = x^2 \\ du = 2x\,dx \\ x = 0 \Rightarrow u = 0,\ x = \sqrt{\ln 2} \Rightarrow u = \ln 2 \end{array}\right] \rightarrow \int_0^{\ln 2} e^u\,du = \left[e^u\right]_0^{\ln 2} = e^{\ln 2} - e^0 = 2 - 1 = 1$$

Problem 2

$$\int \frac{d\theta}{\sqrt{2\theta - \theta^2}} = \int \frac{d\theta}{\sqrt{1-(\theta-1)^2}}\ ;\ \begin{bmatrix} u = \theta - 1 \\ du = d\theta \end{bmatrix} \to \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C = \sin^{-1}(\theta - 1) + C$$

Problem 3

$$\int (\sin 3x \cos 2x - \cos 3x \sin 2x)\, dx = \int \sin (3x - 2x)\, dx = \int \sin x \, dx = -\cos x + C$$

Problem 4

$$\int_{-\pi}^{0} \sqrt{1-\cos^2\theta}\,\mathrm{d}\theta = \int_{-\pi}^{0} |\sin\theta|\,\mathrm{d}\theta; \begin{bmatrix} \sin\theta \le 0 \\ \text{for } -\pi \le \theta \le 0 \end{bmatrix} \rightarrow \int_{-\pi}^{0} -\sin\theta\,\mathrm{d}\theta = [\cos\theta]_{-\pi}^{0} = \cos 0 - \cos(-\pi)$$

$$= 1-(-1) = 2$$

Problem 5

$$\int_{\sqrt{2}}^{3} \frac{2x^3}{x^2-1}\,dx = \int_{\sqrt{2}}^{3}\left(2x + \frac{2x}{x^2-1}\right) dx = \left[x^2 + \ln|x^2-1|\right]_{\sqrt{2}}^{3} = (9+\ln 8) - (2+\ln 1) = 7 + \ln 8$$

Problem 6

$$\int \frac{1-x}{\sqrt{1-x^2}}\,dx = \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{x\,dx}{\sqrt{1-x^2}} = \sin^{-1} x + \sqrt{1-x^2} + C$$

Problem 7

$$\int \frac{\mathrm{dx}}{1+\sin \mathrm{x}} = \int \frac{(1-\sin \mathrm{x})}{(1-\sin^2 \mathrm{x})}\, \mathrm{dx} = \int \frac{(1-\sin \mathrm{x})}{\cos^2 \mathrm{x}}\, \mathrm{dx} = \int (\sec^2 \mathrm{x} - \sec \mathrm{x} \tan \mathrm{x})\, \mathrm{dx} = \tan \mathrm{x} - \sec \mathrm{x} + \mathrm{C}$$

Extra

$\ln x^{(x^x)} = x^x \ln x$ and $\ln (x^x)^x = x \ln x^x = x^2 \ln x$; then, $x^x \ln x = x^2 \ln x \Rightarrow (x^x - x^2)\ln x = 0 \Rightarrow x^x = x^2$ or $\ln x = 0$. $\ln x = 0 \Rightarrow x = 1$; $x^x = x^2 \Rightarrow x \ln x = 2 \ln x \Rightarrow x = 2$. Therefore, $x^{(x^x)} = (x^x)^x$ when $x = 2$ or $x = 1$.