Axion physics in a Josephson junction environment

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Abstract

We show that recent experiments based on Josephson junctions, SQUIDS, and coupled Josephson qubits have a cosmological interpretation in terms of axionic dark matter physics, in the sense that they allow for analogue simulation of early-universe axion physics. We discuss new experimental setups in which SQUID-like axionic interactions in a resonant Josephson junction environment can be tested, similar in nature to recent experiments that test for quantum entanglement of two coupled Josephson qubits. The parameter values relevant for early-universe axion cosmology are accessible with present day's achievements in nanotechnology.

1 Introduction

What do advanced Josephson junction technologies, SQUIDs, coupled Josephson qubits and related superconducting devices used in nanotechnology have in common with the problem of dark matter in the early universe? A lot more than might seem obvious at first sight. One of the major candidates for dark matter in the universe is the axion. The equation of motion of the axion misalignment angle and that of the phase difference in a Josephson junction are identical if the symbols in the mathematical equations are properly re-interpreted. This allows for analogue simulation of early-universe physics using superconducting electronic devices such as SQUIDS and Josephson junctions. It also allows for new experimental setups that test axionic interaction strengths in a Josephson junction environment, similar in nature to recent experiments that test for quantum entanglement of two coupled Josephson qubits. We show in this paper that the parameter values relevant for early-universe axion cosmology are accessible with present day's achievements in nanotechnology. Moreover, we will discuss novel types of SQUID-like interaction states by which axionic dark matter may couple into a given resonant Josephson junction environment. This may pave the way for novel dark matter detectors in the future.

The research fields of superconductivity and cosmology are normally proceeding independently of each other, and the two scientific communities don't know each other — dealing apparently with very different subject areas. But a look at the equations of motions of Josephson junctions, SQUIDS, coupled Josephson qubits and similar superconducting devices used in nanotechnology on the one hand and axionic dark matter on the other indicates that it makes sense to think about common approaches in both areas. The equations of motion are very much the same (with a suitable re-interpretation of the currents involved) and hence it makes sense to develop common approaches and to translate known results from one area (Josephson junctions) into possible results and phenomena for the other area (axion cosmology). This will be worked out in this paper.

There are currently two major candidates for dark matter in the universe, with extensive experimental searches for both of them, WIMPS (weakly interacting massive particles) [1, 2, 3] and axions [4, 5, 6, 7]. In contrast to WIMPS, axions are very light particles. In spite of their small mass, they behave like a very cold quantum gas [5]. There are several experiments that search for axions in the laboratory, using e.g. cavities and strong magnetic fields, which trigger the decay of axions into two microwave photons [5, 6, 8]. These microwave photons are detectable if the cavity resonates with the axion mass. Searches have been unsuccessful so far, but one needs to scan a huge range of cavity frequencies. For this purpose, SQUID amplifiers with very low noise levels are a very useful technological tool to reduce the noise level and to improve the scanning efficiency [8]. These applications of low-level noise SQUID amplifiers for axion searches are very important from an efficiency point of view for cavity experiments, but different from the fundamental physics analogies between axions and Josephson junctions that we want to emphasize in this paper. Novel ideas of experimental axion detection have recently also been presented in [9, 10], based on cold molecules as suitable detectors [9] and analysis of X-rays from the sun [10]. Quasi-axionic particles play also an important role in topological insulators, new materials with exotic metallic states on their surfaces [11, 12, 13, 14]. All this illustrates that it does make sense to look at axion physics in a much broader context than within the original model, which was dealing with the strong CP problem in the standard model of elementary particle physics [15].

The axion is described by a phase angle, the axion misalignment angle. A Josephson junction is also characterized by a phase, namely the phase difference of the macroscopic wave function describing the two superconducting electrodes of the junction. Our major motivation in this paper is the fact that the equation of motion of the axion misalignment angle is identical to that of the phase difference of a resistively shunted Josephson junction, with a suitable re-interpretation of the symbols involved. As a first step, this opens up the theoretical possibility to connect both fields, and to make analogue experiments simulating axion cosmology using superconducting devices in the laboratory. Moreover, and more importantly, this novel approach also opens up the possibility to test for interaction strengths of incoming present-day axionic dark matter in a given resonant Josephson junction environment. The principal idea is that axions may briefly form SQUID-like interaction states in a resonant Josephson environment, before decaying into microwave photons. There are no cosmological constraints on these types of dark matter interactions, since almost all of the matter of the universe is not in the form of Josephson junctions. The only way to either confirm or refute such a hypothesis is by doing laboratory experiments. Suggested future experiments of this type can be easily performed in the laboratory and may ultimately open up the route for novel methods of dark matter detection based on modern nanotechnology.

This paper is organized as follows. In section 2 we point out the equivalence of the field equations of axions and Josephson junctions. In section 3 we show that there is quantitative agreement of the relevant parameters. In section 4 we investigate SQUID-like interaction states of axions and Josephson junctions. Finally, in section 5 the axionic Josephson effect is discussed.

2 Comparing the equations of axion- and Josephson junction physics

Let's compare the mathematics underlying both axions and Josephson junctions. Consider an axion field $a = f_a\theta$, where θ is the axion misalignment angle and f_a is the axion coupling constant. In the early universe, described by a Robertson Walker metric, the equation of motion of the axion misalignment angle θ is

$$\ddot{\theta} + 3H\dot{\theta} + \frac{m_a^2 c^4}{\hbar^2} \sin\theta = 0, \tag{1}$$

neglecting spatial gradients. Here H is the Hubble parameter and m_a denotes the axion mass. The forcing term $\sin\theta$ is due to QCD instanton effects. In a mechanical analogue, the above equation is that of a pendulum in a constant gravitational field with some friction determined by H.

When electric and magnetic fields $\vec{E}$ and $\vec{B}$ are present as well, the axion couples as follows:

$$\ddot{\theta} + 3H\dot{\theta} + \frac{m_a^2 c^4}{\hbar^2} \sin\theta = \frac{g_\gamma}{\pi}\frac{1}{f_a^2} c^3 e^2 \vec{E}\vec{B}. \tag{2}$$

g_γ is a model-dependent dimensionless coupling constant ($g_\gamma = -0.97$ for KSVZ axions, $g_\gamma = 0.36$ for DFSZ axions). The typical parameter ranges that are allowed for dark matter axions are [5, 6]

$$6 \cdot 10^{-6} eV \leq m_a c^2 \leq 2 \cdot 10^{-3} eV \tag{3}$$

and

$$3 \cdot 10^{9} GeV \leq f_a \leq 10^{12} GeV. \tag{4}$$

The product $m_a c^2 f_a$ is of the order $m_a c^2 f_a \sim 6 \cdot 10^{15} (eV)^2$.

Let's now look at resistively shunted Josephson junctions (RSJs) [16, 17]. In the 'tilted-washboard' model the phase difference δ of the macroscopic wave function of the two superconductors satisfies

$$\ddot{\delta} + \frac{1}{RC}\dot{\delta} + \frac{2eI_c}{\hbar C}\sin\delta = 0. \tag{5}$$

Here R is the shunt resistance, C is the capacity of the junction, and I_c is the critical current of the junction. The frequency

$$\omega = \sqrt{\frac{2eI_c}{\hbar C}} \tag{6}$$

is the plasma frequency of the Josephson junction. The product

$$Q := \omega RC \tag{7}$$

is the quality factor of the junction.

If a bias current I is applied to the junction by maintaining a voltage difference V between the two superconducting electrodes, then the equation of motion is

$$\ddot{\delta} + \frac{1}{RC}\dot{\delta} + \frac{2eI_c}{\hbar C}\sin\delta = \frac{2e}{\hbar C}I. \tag{8}$$

Remarkably, the equations of motion of axions and of RSJs are identical provided the following identifications are made in eqs. (2) and (8):

$$3H = \frac{1}{RC} \tag{9}$$

$$\frac{m_a^2 c^4}{\hbar} = \frac{2eI_c}{C} \tag{10}$$

$$\frac{g_\gamma}{\pi f_a^2} c^3 e^2 \vec{E}\vec{B} = \frac{2e}{\hbar C}I. \tag{11}$$

An interesting consequence of this observation is the fact that it is possible to make analogue experiments with RSJs that simulate axion cosmology in the laboratory. To simulate an axion in a certain era of cosmological evolution, one builds up an RSJ with parameters R, C, I_c, I fixed by eqs. (9)-(11). The left-hand side are cosmological parameters, the right-hand side is nanotechnological engineering. Essentially the inverse Hubble parameter H^{-1} (the age of the universe) fixes the product RC, the axion mass fixes the critical current I_c and the axion coupling to external electromagnetic fields $\vec{E}\vec{B}/f_a^2$ is represented by the strength of the bias current I.

3 Coincidence of axion parameters with those of superconducting devices

We now show that there is not only qualitative, but also quantitative coincidence between axion and Josephson junction physics. As worked out below, the numerical values of the parameters for typical axionic dark matter physics and for typical Josephson junction experiments have similar order of magnitude. Let us start with the experiments performed by Koch, Van Harlingen and Clarke in [18]. They built up four different samples of Josephson junctions with parameters values in the range $R \sim 0.075 - 0.77\Omega$, $C \sim 0.5 - 0.81 pF$, $I_c \sim 0.32 - 1.53 mA$. According to eqs. (9)-(11), these experiments of Koch et al. simulate axion-like particles in a very early universe whose age is of the order $H^{-1} = 3RC \sim 10^{-13} - 10^{-12} s$. The axion mass is in the range $0.98 \cdot 10^{-3} - 1.58 \cdot 10^{-3}$ eV. This simulated axion mass is just at the upper end of what is expected for dark matter axions, see eq. (3).

The recent experiment by Steffen et al. [19], which tests for quantum entanglement of coupled Josephson qubits, is classically well described by an RSJ model where the product $3RC$ is of the order $H^{-1} = 3RC = 1.2 \cdot 10^{-6} s$ [20]. This experiment thus simulates the dynamics of weakly coupled axions *after* the QCD phase transition in the early universe, which took place during an epoch where $H^{-1} \sim 10^{-8} s$. It corresponds to an axion mass of $m_a c^2 = \hbar\omega = 3.3 \cdot 10^{-5} eV$, much smaller than for the Koch et al. experiment [18] but within the range expected for dark matter axions, see eq. (3).

It is interesting to see that recent experiments [19, 20, 21, 22, 23, 24] simulate axions with masses in the entire range of what is interesting from a dark matter point of view. The experiments of Penttillae et al. [21], dealing with superconductor-insulator phase transitions, correspond to $m_a c^2 = 1.32 \cdot 10^{-4} eV$. Nagel et al. [22] deal with negative absolute resistance effects in Josephson junctions, these experiments formally have $m_a c^2 = 2.83 \cdot 10^{-5} eV$. Superconducting atomic contacts [23] correspond to even smaller axion masses, namely $m_a = 6.7 \cdot 10^{-6} eV$, at the lower end of what is allowed in eq. (3). Two-dimensional Josephson arrays, as built up in [24], correspond to arrays of coupled axions with $m_a c^2$ in the range $6.62 \cdot 10^{-5} - 1.52 \cdot 10^{-4} eV$. All these recent experiments are within the range of axion masses that are of interest from a dark matter point of view. They can thus be regarded as realistic analogue experiments simulating an axionic dark matter environment in the early universe.

The main idea of this paper is to go a step further. If experiments of the type mentioned above simulate a kind of realistic axionic dynamics, could these experiments then be used to detect current day incoming dark matter axions, by triggering their decay in a resonant environment? This will be worked out in the following sections.

4 Possible interaction mechanism between axions and Josephson junctions

Consider a RSJ which has a plasma frequency $\omega_p = \sqrt{\frac{2eI_c}{\hbar C}}$ close to the dark-matter axion mass m_a, according to eq. (10), and that is driven by an external bias current $I > I_c$. Free axion quanta correspond to small nearly-harmonic oscillations of the misalignment angle θ, in accordance with eq. (1). Suppose such an axion enters a Josephson junction that has similar parameters as the entering axion. Then this basically means we have a system of two Josephson junctions, the second one represented by the entering axion. The axion is very cold and its potential is given by a tilted washboard potential. Thus its phase variable may couple into a given Josephson environment in a SQUID-like configuration. This is illustrated in Fig. 1.

For any Josephson junction, a given numerical value of a phase difference $\Delta\varphi$ on its own does not have physical meaning since it is not a gauge-invariant quantity (see, e.g., [17]). Rather, what is of physical relevance is the gauge-invariant phase difference

$$\delta = \Delta\varphi - \frac{2\pi}{\Phi_0} \int \vec{A} d\vec{s}, \tag{12}$$

where the integration path is from one electrode of the RSJ to the other ($\vec{A}$ is the vector potential under consideration and $\Phi_0 = \frac{h}{2e}$ is the flux quantum). The term $\int \vec{A} d\vec{s}$ has profound consequences for the physics of Josephson junctions and SQUIDS if magnetic fields are applied.

The gauge-invariant phase of the Josephson-axion-SQUID of Fig. 1 must be single-valued. Putting a closed integration path that passes the weak links as well as the interior of the superconductor, and using the same line of arguments as for ordinary SQUIDS [17], one obtains

$$\delta - \theta = 2\pi \frac{\Phi}{\Phi_0} \mod 2\pi. \tag{13}$$

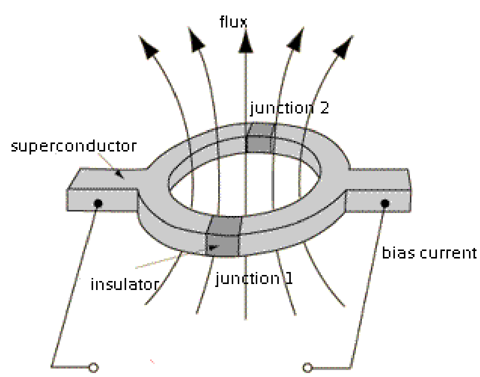

Figure 1: A Josephson junction (junction 1) and an axion (formally represented by junction 2) interacting in a SQUID -like configuration. The supercurrent I flowing through junction 1 simulates to the synchronized axion the formal existence of a very large product $\vec{E} \cdot \vec{B}$ given by eq. (11). Thus the axion immediately decays into two microwave photons.

Here Φ is the magnetic flux enclosed by the SQUID. If the flux Φ is close to zero or given by integer multiples of Φ_0, then the above condition simply implies

$$\delta = \theta, \tag{14}$$

i.e. the two phases synchronize.

Classically, the coupling between two Josephson junctions with shunt resistance R, capacity C, and inductivity L is described by the following coupled differential equations [20]:

$$\begin{aligned} \ddot{\delta} + \frac{1}{RC}\dot{\delta} + \omega^2 \sin\delta &= \gamma_x(\ddot{\theta} - \ddot{\delta}) + \frac{1}{CL}(\delta + 2\pi M_1) \\ \ddot{\theta} + \frac{1}{RC}\dot{\theta} + \omega^2 \sin\theta &= \gamma_x(\ddot{\delta} - \ddot{\theta}) + \frac{1}{CL}(\theta + 2\pi M_2) \end{aligned} \tag{15}$$

Here δ is the phase difference of the first junction, θ that of the second junction. $M_i = \Phi_i/\Phi_0$ is the normalized flux enclosed by junction i $(i = 1, 2)$, and $\gamma_x = C_x/C$ is a small dimensionless coupling constant, assuming both junctions are capacitively coupled by a capacity C_x. For example, in the experiments of Steffen et al. dealing with coupled Josephson qubits [19] one has $\gamma_x = 2.3 \cdot 10^{-3}$. Typically, the damping term proportional to $\dot{\delta}$ and $\dot{\theta}$ is neglected in the theoretical treatment of these types of experiments [20]. The above classical equations of motion describe quite well the experimentally observed phenomena [19, 20].

In our case the phase δ describes an ordinary Josephson junction and the phase θ an axion that passes through this Josephson junction. A very simple coupling scheme is given by

$$\begin{aligned} \ddot{\delta} + \frac{1}{RC}\dot{\delta} + \omega^2 \sin\delta &= \gamma_x(\ddot{\theta} - \ddot{\delta}) \\ \ddot{\theta} + 3H\dot{\theta} + \frac{m_a^2 c^2}{\hbar^2} \sin\theta &= \gamma_x(\ddot{\delta} - \ddot{\theta}). \end{aligned} \tag{16}$$

This corresponds to the case $L \to \infty$ in eq. (15).

If the axion mass is at resonance with the Josephson plasma frequency, $m_a c^2 = \hbar\omega$, then synchronization effects of the phases δ and θ will occur if γ_x is not too small, just as they occur for coupled Josephson qubits [19, 20]. If the axion couples to fluxes similar as in eq. (15), then this is described by a small additional self-interaction potential given by $V(a) =$

$-\frac{1}{CL}(\frac{1}{2}a^2 + 2\pi M_2 f_a a)$. Quantum mechanically, one could even speculate on the formation of entangled states between axions and Josephson qubits.

Given the quantitative agreement between the parameters of axion physics and Josephson junction physics outlined in section 3, one might hope that for axions the coupling γ_x is again of similar order of magnitude as in current nanotechnological experiments, provided the Josephson plasma frequency is close to the axion mass. This can be experimentally tested.

There are no astronomical constraints on the size of γ_x since almost all of the matter in the universe is not in the form of Josephson junctions. Axionic dark matter may look completely 'dark' in the universe as a whole but not at all 'dark' in special superconducting devices designed by mankind, due to SQUID-like interactions. The only way to constrain γ_x is to scan a range of plasma frequencies and look for the possible occurence or non-occurence of universal resonance effects, produced by axions of the dark matter halo that hit terrestrial Josephson junction experiments. The intensity of this effect might display small yearly modulations, just similar as in the DAMA/LIBRA experiments [1]. What corresponds to tuning the cavity frequency in the experiments [8] would correspond to tuning the plasma frequency ω in these new types of nanotechnological dark matter experiments.

5 Axionic Josephson effect

As an application of our theoretical treatment in the previous section, let us now discuss the analogue of the Josephson effect for axions, similar in spirit to what was experimentally observed for BEC in [25]. A Josephson junction biased with voltage V generates Josephson radiation with frequency

$$\hbar\omega_J = 2eV. \tag{17}$$

For such a biased junction the phase δ grows linearly in time, i.e.

$$\delta(t) = \delta(0) + \frac{2eV}{\hbar}t. \tag{18}$$

The relation between bias current I and applied voltage V is

$$V = R\sqrt{I^2 - I_c^2} \approx RI \quad \text{for } I >> I_c. \tag{19}$$

Josephson oscillations set in if

$$I > I_c, \tag{20}$$

i.e. the bias current I must be larger than the critical current I_c of the junction. In the mechanical analogue, the pendulum rotates with large kinetic energy.

According to eq. (2) and (8), the axion misalignment angle θ will also start to increase linearly in time if it is being forced by very strong products of $\vec{E}$ and $\vec{B}$ fields. So from a formal mathematical point of view, an axionic Josephson effect is possible. The axionic Josephson frequency is given by

$$\hbar\omega_J = 2eV \approx 2eRI = \frac{g_\gamma}{\pi}\frac{1}{f_a^2}c^3\vec{E}\cdot\vec{B}, \tag{21}$$

where in the last step eq. (11) was used. Condition (20) translates to

$$e^2\hbar^2\vec{E}\vec{B} > f_a^2 m_a^2 c = \Lambda^4 c \tag{22}$$

QCD-inspired models of axions fix Λ to be about 78 MeV [4]. Strong magnetic fields in the laboratory correspond to about 10 Tesla, and strong electric fields to about $10^9 V/m$. This gives $\vec{E}\cdot\vec{B} \approx 10^{10} VT/m$. One can easily check that under normal laboratory conditions one cannot produce stationary $\vec{E}$ and $\vec{B}$ fields of sufficient strength to satisfy eq. (22).

However, there is another interesting possibility how we can briefly induce axionic Josephson oscillations in the lab. This is based on the SQUID-like interaction mechanism discussed in section 4. Remember that the phase of a SQUID formed out of a Josephson junction and a passing axion should be gauge-invariant, provided the axion mass is at resonance with the plasma frequency. This led us to derive eq. (13), meaning that the phase δ of the Josephson junction and the phase θ of the axion synchronize. For a biased RSJ performing Josephson oscillations of frequency ω_J, synchronization means that

$$\delta(t) = \delta(0) + \omega_J t \tag{23}$$

induces

$$\theta(t) = \theta(0) + \omega_J t \tag{24}$$

for the axion. According to eq. (11), this means that the axion formally sees a huge product field $\vec{E}\cdot\vec{B}$. The huge (virtual) magnetic field will make it immediately decay into two microwave photons. The microwave photons produced by axion decay have the frequency of the Josephson radiation and produce distortions in the $I-V$ curve. They can be potentially measured in form of Shapiro steps (Shapiro steps are well-known step-like structures in the $I - V$ curves of irradiated Josephson junctions [26]). Our theoretical idea thus opens up the possibility to develop new detectors for axionic dark matter[1].

6 Conclusion

Let us conclude. In this paper we have discussed the possibility that axions could interact with Josephson junctions by briefly forming SQUID-like interaction states. Josephson junctions, SQUIDS, and spatially extended arrays of these superconducting devices can nowadays be built for a wide range of different parameters R, C, I_c, and it is very easy to tune the bias current I to any value of interest. It is also very easy to adjust the plasma frequency of a Josephson junction to any value of interest. It is thus possible to build up a suitable resonant environment that could help to detect incoming dark matter axions.

There is the prospect of developing new generations of detectors for dark matter axions that search for possible resonance effects and phase synchronization if the Josephson plasma frequency is close to the axion mass. In this way the size of the coupling γ_x between axions and a given Josephson environment can be experimentally constrained. As shown in section 3, the relevant dark matter mass parameter range is accessible by modern technological developments in nanotechnology. An obvious advantage of these types of experiments is that the formal existence of extremely large products of electric and magnetic field strengths $\vec{E}\vec{B}$ acting on the axion can be simulated by a very simple experimental setup, an easily tunable bias current I, assuming that some axions hitting the Josephson junction will synchronize their phase due to a SQUID-like interaction. This effect may be systematically tested in future experiments.

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