

Axiomatic approach to the cosmological constant

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A theory of the cosmological constant Λ is currently out of reach. Still, one can start from a set of axioms that describe the most desirable properties a cosmological constant should have. This can be seen in certain analogy to the Khinchin axioms in information theory, which fix the most desirable properties an information measure should have and that ultimately lead to the Shannon entropy as the fundamental information measure on which statistical mechanics is based. Here we formulate a set of axioms for the cosmological constant in close analogy to the Khinchin axioms, formally replacing the dependency of the information measure on probabilities of events by a dependency of the cosmological constant on the fundamental constants of nature. Evaluating this set of axioms one finally arrives at a formula for the cosmological constant given by $\Lambda = \frac{1}{\hbar^4} G^2 \left(\frac{m_e}{\alpha_{el}} \right)^6$, where G is the gravitational constant, m_e the electron mass, and α_{el} the low energy limit of the fine structure constant. This formula is in perfect agreement with current WMAP data. Our approach gives physical meaning to the Eddington-Dirac large number hypothesis and suggests that the observed value of the cosmological constant is not at all unnatural.

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I. INTRODUCTION

The cosmological constant problem is probably one of the most fundamental problems in physics that so far has resisted any attempt of solution [1]. When looking through the large amount of literature on the cosmological constant Λ and the associated cosmological constant problem, one statement is found quite regularly: The observed value of the cosmological constant (or dark energy density) that is suggested by WMAP and other astronomical observations [2, 3] is regarded by most physicists to be rather unnatural and surprising, and some people from the anthropic school even regard it to be unexplainable. From a quantum field theory point of view one would have a priori expected a value of vacuum energy density $\rho_{vac} = \frac{c^4}{8\pi G} \Lambda$ given by something of the order m_{pl}^4 (in units where $\hbar = c = 1$), since the Planck mass m_{pl} is a suitable cutoff scale for vacuum fluctuations where quantum field theory needs to be replaced by something else. So for a quantum field theorist the observed value of the cosmological constant is surprisingly small. Astrophysicists, on the other hand, are facing a rather large value of Λ in the Λ CDM model as compared to observable mass densities. This means that the current universe is dominated by vacuum energy, whereas a priori most astrophysicists would have probably expected dark energy to play a less pronounced role, so for them Λ is surprisingly large. In supersymmetric theories, and in particular superstring theory, the most natural value of the cosmological constant is zero, and again it is not clear

how to obtain a small positive value at the current time by a 'natural' mechanism. This has led to models based on anthropic considerations, which give up the idea of a single universe and regard Λ as a random variable whose value (by construction of the anthropic argument) can never be explained, just probabilistic statements can be given for an ensemble of 'multiverses' [4].

Given all these controversies and mysteries surrounding the cosmological constant, it is perhaps worth to go back to the basics and to ask ourselves how natural or unnatural the observed value really is. Since the development of a full theory of the cosmological constant is currently out of reach, we will start by formulating a set of axioms that describe the most desirable properties a cosmological constant should have. This can be seen in analogy to the set of Khinchin axioms [5, 6] in information theory that describe the most desirable properties an information measure should have. It is well-known that from the Khinchin axioms one can uniquely derive the Shannon entropy, on which the entire mechanism of statistical mechanics is founded (see any textbook on the subject, e.g. [7]). Similarly, we will formulate suitable axioms for a cosmological constant. The principal idea underlying this approach is that ultimately the cosmological constant is expected to be part of a unified theory of quantum gravity and the standard model of electroweak and strong interactions. It will thus potentially depend on fundamental constants of nature. The axioms that we formulate deal with possible dependencies on these fundamental parameters.

Roughly speaking, the physical contents of these axioms is as follows: The cosmological constant should only depend on fundamental parameters of nature (rather than irrelevant parameters), it should be bounded from below, it should depend on the relevant fundamental con-

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stands in the simplest possible way, and the dependence should be such that one obtains scale invariance of the universe under suitable transformations of the fundamental parameters that leave the physics invariant. We will point out that these 4 axioms for the cosmological constant are very similar in style to the 4 Khinchin axioms that ultimately underlie the foundations of statistical mechanics. Amazingly, out of the 4 axioms a formula for Λ can be derived that is in excellent agreement with current observations. This formula is given by

$$\Lambda = \frac{1}{\hbar^4} G^2 \left(\frac{m_e}{\alpha_{el}} \right)^6, \quad (1)$$

where G is the gravitational constant, m_e the electron mass, and α_{el} the low-energy limit of the fine structure constant.

It turns out that the result (1) of our derivation can be interpreted as a particular form of the Eddington-Dirac large number hypothesis, connecting cosmological parameters and fundamental constants [8, 9, 10, 11, 12], in a form previously advocated by Nottale [10, 11], based on previous work by Zeldovich [13]. Still our derivation is very different from Nottale's original approach, since we do not need any assumption on 'scale relativity' [11]. Rather, our method is much more related to an information-theoretic approach (similar as in statistical mechanics) and gives new physical meaning to this kind of approach. The above value of the cosmological constant is singled out as a kind of optimum value that is consistent with the axioms.

The vacuum energy density (dark energy density with equation of state $w = -1$) that follows from our axiomatic approach is given by

$$\rho_{vac} = \frac{c^4}{8\pi G} \Lambda = \frac{1}{8\pi} \frac{c^4}{\hbar^4} G \left(\frac{m_e}{\alpha_{el}} \right)^6. \quad (2)$$

According to the 4 axioms that we will formulate in the following sections, eq. (2) yields a kind of an optimum value of the vacuum energy in the universe, according to criteria set out by the axioms. Numerically, this formula yields the prediction

$$\rho_{vac} = (4.0961 \pm 0.0006) GeV/m^3 \quad (3)$$

The current astronomical measurements provide evidence for a dark energy density of about

$$\rho_{dark} = (3.9 \pm 0.4) GeV/m^3. \quad (4)$$

We thus conclude that the observed value of the cosmological constant is not at all unnatural, but derivable from a set of suitable axioms that make physical sense.

This paper is organized as follows. In section 2 we briefly recall the Khinchin axioms, in order to make this paper self-contained for readers that do not have an information theory background. In section 3 we formulate our 4 axioms for the cosmological constant and point out

the analogy with the Khinchin axioms. In section 4 we derive the above formula for ρ_{vac} from the axioms. In section 5 we point out that our axiomatic approach gives physical meaning to the Eddington-Dirac large number conjecture. Our concluding remarks are given in section 6.

Throughout this paper our notion of ρ_{vac} means the observable (physically relevant) vacuum energy density, which should be distinguished from any bare (unmeasurable) contributions, see e.g. [14] for a discussion of this subtlety.

II. THE KHINCHIN AXIOMS

Khinchin [5] formulated four axioms that describe the most desirable properties an information measure I should have. These 4 axioms uniquely fix the functional form of the Shannon information and are extremely important for the mathematical foundations of statistical mechanics. Let us recall these 4 axioms as well as their physical meaning. Later, we will formulate analogous axioms for the cosmological constant.

A1 'fundamentality'

$$I = I(p_1, \dots, p_W) \quad (5)$$

That is to say, an information measure I only depends on the probabilities p_i of the events under consideration ($i = 1, \dots, W$) and nothing else. It should not depend on any other irrelevant or non-fundamental quantities.

A2 'boundedness'

$$I(W^{-1}, \dots, W^{-1}) \leq I(p_1, \dots, p_W) \quad (6)$$

This means the information measure I takes on an absolute minimum for the uniform distribution (W^{-1}, \dots, W^{-1}), every other probability distribution has an information contents that is larger or equal to that of the uniform distribution. Clearly, this implies there is a lower bound for I . We may thus call this the 'axiom of boundedness'.

A3 'simplicity'

$$I(p_1, \dots, p_W) = I(p_1, \dots, p_W, 0) \quad (7)$$

This means the information measure I should not change if the sample set of events is enlarged by another event that has probability zero. Clearly, one can make the description of a given model as complicated as one like, but this axiom advocates the simplest description, where irrelevant events with probability zero are excluded. We may call this the 'axiom of simplicity'.

A4 'invariance'

$$I(\{p_{ij}^{I,II}\}) = I(\{p_i^I\}) + \sum_i p_i^I I(\{p^{II}(j|i)\}) \quad (8)$$

This axiom is slightly more complicated and requires a longer explanation. The axiom deals with the composition of two systems I and II (not necessarily independent). The probabilities of the first system are p_i^I , those

of the second system are p_j^{II} . The joint system I, II is described by the joint probabilities $p_{ij}^{I,II} = p_i^I p^{II}(j|i)$, where $p^{II}(j|i)$ is the conditional probability of event j in system II under the condition that event i has occurred in system I. $I(\{p^{II}(j|i)\})$ is the conditional information of system II formed with the conditional probabilities $p^{II}(j|i)$, i.e. under the condition that system I is in state i .

The meaning of the above axiom is that it postulates that the information measure should be independent of the way the information is collected. We can first collect the information in the subsystem II, assuming a given event i in system I, and then sum the result over all possible events i in system I, weighting with the probabilities p_i^I .

For the special case that system I and II are independent the probability of the joint system factorizes as

$$p_{ij}^{I,II} = p_i^I p_j^{II}, \quad (9)$$

and only in this case, axiom 4 reduces to the rule of additivity of information for independent subsystems:

$$I(\{p_{ij}^{I,II}\}) = I(\{p_i^I\}) + I(\{p_j^{II}\}) \quad (10)$$

Apparently, the above axiom deals with the fact that there is some invariance of the information measure if a description in terms of other probabilities (namely the conditional ones) associated with other subsystems is chosen. In a more abstract way we may write

$$I(\{\tilde{p}_i\}) = \tilde{I}(\{p_i\}) \quad (11)$$

meaning that there is a suitable scale transformation (denoted by $\tilde{}$) in the space of probabilities and information measures that leaves the physical contents invariant. We may thus call the 4th axiom the 'axiom of invariance'.

One can now easily show that the functional form of the Shannon information

$$I(p_1, \dots, p_W) = \sum_i p_i \log p_i \quad (12)$$

follows uniquely from axioms A1–A4 (see any textbook on this topic, for example [7]). The fourth axiom actually is the most important one. While there are many different information measures satisfying A1–A3, the specific form of the Shannon information is crucially determined by A4 [6].

III. AXIOMS FOR THE COSMOLOGICAL CONSTANT

We now proceed to axioms for the cosmological constant, which will turn out to share a certain analogy with the Khinchin axioms. The role of the information measure is now formally played by Λ , and the dependence of I on the probabilities p_i is replaced by the dependence of

Λ on the fundamental constants of nature, such as coupling constants α_i , masses m_i , and mixing angles s_i . Let us now formulate the following set of axioms B1–B4:

B1 'fundamentality'

$$\Lambda = \Lambda(\{\alpha_i\}, \{m_i\}, \{s_i\})$$

The cosmological constant depends on fundamental constants of nature only. There is no dependence on non-fundamental parameters.

B2 'boundedness'

$$0 < \Lambda$$

The cosmological constant is bounded from below. The trivial solution $\Lambda = 0$ is not allowed.

B3 'simplicity'

$$\Lambda(\{\alpha_i\}, \{m_i\}, \{s_i\}) = \Lambda(\{\alpha_i\}, \{m_i\}, \{s_i\}, \{c_i\})$$

The cosmological constant is given by the simplest possible formula consistent with the other axioms, avoiding irrelevant non-universal prefactors or dependencies on irrelevant parameters c_i .

B4 'invariance'

$$\Lambda(\{\tilde{\alpha}_i\}, \{\tilde{m}_i\}, \{\tilde{s}_i\}) = \tilde{\Lambda}(\{\alpha_i\}, \{m_i\}, \{s_i\})$$

A cosmological constant formed with potentially different values of fundamental parameters leaves the large-scale physics of the universe scale invariant.

As we can see, axiom B1 is basically the same as Khinchin axiom A1, formally replacing the information measure I by the cosmological constant Λ and the dependence on the probabilities p_i of events by the dependence of Λ on the fundamental constants of nature. Axiom B2, just like axiom A2, states that there is a lower bound on the cosmological constant, which in our case is taken to guarantee that Λ is bounded away from zero, thus excluding the trivial solution. In contrast to A2, we need a strict inequality in B2. Axiom B3, just like A3, advocates that the simplest possible description is the physically relevant one. Whereas A3 excludes irrelevant events (those with probability 0), in a similar way B3 excludes irrelevant non-fundamental constants from influencing Λ . Finally, the most important and restrictive axiom is axiom B4. Similar as A4, which yields the total information I obtained by using different types of probabilities in different subsystems, B4 deals with a description of the cosmological constant using potentially different constants of nature in a different universe. Similar to A4, B4 postulates that the large-scale physics should be scale invariant under this transformation process of fundamental parameters. In other words, in relative terms the effect of the cosmological constant as compared to other interaction strengths should be unchanged.

IV. DERIVATION OF THE COSMOLOGICAL CONSTANT FROM THE AXIOMS

From B2 it follows that the cosmological constant has a nontrivial positive value. From B1 and B3 it follows that this value can be written as the simplest possible combination of relevant fundamental constants of nature that is consistent with B4.

As for the Khinchin axioms, the 4th axiom for the cosmological constant is also the most restrictive one. This axiom is dealing with the physical effects that the cosmological constant has in an evolving universe and puts it into relation with the other interactions. B4 says that the large scale physical effect of the cosmological constant should be invariant if relevant fundamental parameters change to different values. In other words, the *relative* effect of the cosmological constant as compared to the other relevant large-scale physics should be the same under fundamental parameters transformations. We note that the cosmological constant acts on a large scale and hence it is unlikely that it is influenced by the fundamental parameters of strong and weak interactions, which are relevant for short-range physics only. If at all, it should be influenced by interactions that have a long range: gravity and electromagnetism.

Of physical meaning for the evolution of the universe is really the vacuum energy density $\rho_{vac} = \frac{c^4}{8\pi G}\Lambda$ associated with Λ . In the following we will derive in three steps the simplest form of vacuum energy density that is consistent with axiom B4. In the first step, we apply B4 to gravitational physics. In the second step, a dimensional argument is given. Finally, in the third step we take into account electromagnetic interaction processes. Putting together all 3 steps we finally end up with a concrete formula for ρ_{vac} .

A. Step 1: Gravitational scale invariance

Consider an arbitrary spatial volume V of the universe at an arbitrary time, which (at a late stage) may contain galaxies, stars, dust etc. We denote the point masses contained in this volume by m_i , $i = 1, 2, 3, \dots$. The total gravitational energy density in this volume V is

$$\rho_G = -\frac{G}{V} \sum_{i,j} \frac{m_i m_j}{r_{ij}}, \quad (13)$$

where r_{ij} is the distance between masses m_i and m_j and the sum runs over all indices i, j with $i > j$. At the same time there is also constant vacuum energy density ρ_{vac} in this region of the universe. Now consider a fundamental parameter transformation in the universe which gives the gravitational constant G a new value. We keep all distances r_{ij} and masses m_i as they are but formally change the gravitational constant

$$G \rightarrow \Gamma G. \quad (14)$$

For example, $\Gamma = 2$. Then the gravitational energy density ρ_G also doubles:

$$\rho_G \rightarrow \Gamma \rho_G \quad (15)$$

Keeping the original vacuum energy density ρ_{vac} would clearly change the relative size of ρ_G and ρ_{vac} . Scale

invariance of the universe as a whole is only achieved if at the same time ρ_{vac} is also transformed as

$$\rho_{vac} \rightarrow \Gamma \rho_{vac} \quad (16)$$

Only in this case the ratio of gravitational energy density and vacuum energy density stays the same and the universe remains scale invariant. In other words, the postulate B4 of scale invariance of the universe under the fundamental parameter transformation (14) implies that the vacuum energy density must be proportional to G :

$$\rho_{vac} \sim GX \quad (17)$$

Here X denotes something that is so far unknown but independent of the gravitational constant G . Eq. (17) already represents an important step which strongly restricts the possible dependencies of ρ_{vac} on the fundamental constants of nature, as we shall see in the following.

B. Step 2: A dimensional argument

Let us now discuss in more detail the form of the quantity X in eq. (17). For dimensionality reasons, in units where $\hbar = c = 1$, we must multiply $G = 1/m_{pl}^2$ by some mass scale to the power 6 in order to get an energy density, which in these units has the dimension mass to the power 4. If we allow for an arbitrary dimensionless proportionality factor A , we may choose this reference mass scale to be the electron mass m_e , which surely is a fundamental constant, and write

$$\rho_{vac} = AGm_e^6 \quad (18)$$

We now return to units where \hbar and c are measured in SI units. In this case a prefactor c^4/\hbar^4 is needed to give ρ_{vac} the correct dimension of an energy per volume:

$$\rho_{vac} = A \frac{c^4}{\hbar^4} G m_e^6 \quad (19)$$

The arbitrary dimensionless proportionality factor A can be expressed rather arbitrarily in terms of the fine structure constant α_{el} as $A = \frac{1}{8\pi} \alpha_{el}^\eta$ if we allow η to be an arbitrary real number. Hence

$$\rho_{vac} = \frac{1}{8\pi} \frac{c^4}{\hbar^4} G m_e^6 \alpha_{el}^\eta \quad (20)$$

Since η is arbitrarily chosen, not very much is gained so far. However, we will now show in the third and final step that axioms B1, B3, and B4 applied to electromagnetic interaction processes in the early universe will fix the so far arbitrary real number η .

C. Step 3: Electromagnetic scale invariance

After recombination, almost all free charges in the universe vanish due to the formation of atoms which are electrically neutral on a large scale. Hence, in contrast to

gravitational interaction, electromagnetic interaction is irrelevant on large scales after recombination. The large-scale physics of the universe, however, is dominated by electromagnetic interaction processes *before* recombination, where lots of free charges exist that interact with photons. The relevant electromagnetic scattering process before recombination is Thomson scattering. The total cross section σ_T for Thomson scattering of particles of mass m and charge Q is given by

$$\sigma_T = \frac{8\pi}{3} \left(\alpha_{el} Q^2 \frac{\hbar}{mc} \right)^2. \quad (21)$$

Note that this cross section is dominated by the lightest charged particles, i.e. electrons (charge $Q = -1$, mass $m = m_e$). Compared to electrons, Thomson scattering of heavy particles such as μ , τ or protons is clearly negligible in the early universe, due to the higher mass involved. For electrons, we may write

$$\sigma_T = \frac{8\pi}{3} r_e^2 \quad (22)$$

where

$$r_e = \alpha_{el} \frac{\hbar}{m_e c} \quad (23)$$

is the classical electron radius. Thomson scattering of electrons is the most important scattering process in the early universe, and is thought to be responsible for the linear polarization of the cosmic microwave background. It dominates the large-scale physics of the early universe and hence is *the* electromagnetic process that, if any, should be describable in a scale invariant way relative to the energy scale set by the cosmological constant, according to axiom B4.

Let us now consider a fundamental parameter transformation of the following form:

$$\alpha_{el} \rightarrow \Gamma \alpha_{el} \quad (24)$$

$$m_e \rightarrow \Gamma m_e \quad (25)$$

For example, $\Gamma = 2$. The above simultaneous transformation of the electromagnetic parameters α_{el} and m_e leaves the physics of Thomson scattering invariant, since σ_T only depends on the ratio α_{el}/m_e . Relative to this, we want the cosmological constant to stay invariant as well under the above transformation, because otherwise the relative size of the inverse cosmological constant as compared to the Thomson cross section would change, thus violating axiom B4, which requires scale invariance of the universe as a whole (note that Λ^{-1} and σ_T have the same dimension, namely length squared, hence the two quantities are directly comparable). Invariance under the above simultaneous transformation with Γ means that Λ , respectively ρ_{vac} , must be a function of the ratio m_e/α_{el} rather than a function of m_e and α_{el} on their

own. But this fixes the parameter η in eq. (20) to be $\eta = -6$. We thus arrive at the final result

$$\rho_{vac} = \frac{1}{8\pi} \frac{c^4}{\hbar^4} G \left(\frac{m_e}{\alpha_{el}} \right)^6. \quad (26)$$

One remark is still at order. The factor $1/8\pi$ in eq. (26) was introduced by convention and it could still be in principle something else. However, here axiom B1 and B3 help which state that the cosmological constant should be a function of the fundamental constants of nature only, in the simplest possible way, and avoiding arbitrary prefactors for which there is no physical reason. The prefactor $1/8\pi$ for the vacuum energy ρ_{vac} in eq. (26) is precisely chosen in such a way that the cosmological constant

$$\Lambda = \frac{8\pi G}{c^4} \rho_{vac} = \frac{1}{\hbar^4} G^2 \left(\frac{m_e}{\alpha_{el}} \right)^6 \quad (27)$$

does not have any prefactor. So this choice is indeed the simplest possible one, in agreement with axiom B3 of simplicity.

V. CONNECTION WITH EDDINGTON-DIRAC LARGE NUMBER HYPOTHESIS

Using $G = \hbar c/m_{pl}^2$ one can easily check that eq. (27) can be written in the equivalent dimensionless form

$$\alpha_{el} \frac{m_{pl}}{m_e} = \left(\frac{\Lambda^{-1/2}}{l_{pl}} \right)^{\frac{1}{3}}, \quad (28)$$

where $l_{pl} = \hbar/m_{pl}c$ is the Planck length. This can be regarded as an Eddington-Dirac large number hypothesis, since the equation connects cosmological parameters with standard model parameters [8, 9]. Eq. (28) has been written down previously by Nottale [10]. On both sides of the above equation one has two very large numbers. So far these types of large-number relations have often been regarded as being just some type of numerical coincidence. But here we see that the occurrence of such large numbers that coincide is not at all unnatural. Rather, all this follows in a rather straightforward way from our set of axioms.

One may ask why to take the power $1/3$ in the above equation and not some other power. If no theory is available, then this power could in principle be chosen in an arbitrary way. However, the power $1/3$ follows in a stringent and logically consistent way out of our axiomatic approach. In fact, the power $1/3$ in eq. (28) is equivalent to the power 6 of m_e in eq. (26), and this power was derived in the previous section from the postulate of gravitational scale invariance and using our dimensional argument. We see that the above Eddington-Dirac large number relation arises in quite a natural way out of a set of axioms B1–B4 that do make physical sense and that in a way describe the most desirable properties a cosmological constant should have when compared with

the other relevant processes in an evolving universe. In this way our axiomatic approach presented here has given physical meaning to this relation, and makes it plausible that there is more to this relation than just a numerical coincidence of some large numbers.

An interesting aspect is the fact that the relation (27) uses fundamental parameters such as $\hbar, G, m_e, \alpha_{el}$ that are all known with very high precision. Thus this relation allows for an extremely precise prediction of the value of the cosmological constant Λ , by far more precise than the present cosmological observations can confirm:

$$\Lambda = (1.36284 \pm 0.00028) \cdot 10^{-52} m^{-2} \quad (29)$$

The above value can be used to precisely fix the relevant energy scale in other, more advanced microscopic models of dark energy (e.g. [15, 16]).

VI. CONCLUSION

In this paper we have formulated a set of axioms that in a sense describe the most desirable physical properties a cosmological constant should have. This set of axioms can be seen in analogy to the set of Khinchin axioms, which describe the most desirable properties an information measure should have. The Khinchin axioms uniquely lead to the Shannon information measure, which lies at the root of the mathematical foundations of statistical mechanics and thermodynamics. In a similar way, our set of axioms leads to a formula for the cosmological constant that is equivalent to a particular type of Eddington-Dirac large number hypothesis, in a form previously advocated by Nottale. Our approach gives physical meaning to this formula, which so far was only regarded to be a numerical coincidence. The agreement of our derived formula with the WMAP measurements of dark energy density in the universe is amazing, given the fact that *a priori* our set of physically reasonable axioms does not know anything about supernovae measurements and could have led to a completely different vacuum energy density prediction, different from the observed one by many orders of magnitude.

The fact that techniques from information theory (Khinchin-like axioms for the cosmological constant) lead

to apparently successful predictions of the numerical value of Λ opens up new ways to deal with the cosmological constant problem. In fact, tools from information theory and thermodynamics have proved to be useful for gravitational physics in the past as well, the main example being the thermodynamics of black holes. Our approach here suggests that similar information theoretic tools can be useful to derive concrete predictions for the observed dark energy density in the universe, i.e. eq. (3).

Our approach certainly does not provide any microscopic theory underlying the cosmological constant. This must be the task of future work—our work here is just based on information-like techniques as an effective theory. However, again we are reminded of the foundations of statistical mechanics and thermodynamics: These theories work perfectly, based on maximizing the Shannon entropy subject to suitable constraints. The Shannon entropy is identified with the physical Boltzmann-Gibbs entropy and its functional form directly follows from the Khinchin axioms A1–A4. Similarly, the cosmological constant $\Lambda(\{\alpha_i\}, \{m_i\}, \{s_i\})$ in this paper is identified as the source of measurable dark energy in the universe and its functional dependence on the fundamental parameters follows from the axioms B1–B4.

So far nobody has succeeded to provide a rigorous derivation of statistical mechanics starting from the underlying microscopic equations. Rather, one usually starts from the Khinchin axioms, gets from them the Shannon entropy and then proceeds to statistical mechanics and thermodynamics by optimization of the Shannon entropy. This method works perfectly but a deeper understanding of the microscopic foundations of statistical mechanics is not provided. Maybe the cosmological constant awaits a similar fate: Its numerical value (the observed dark energy density in the universe) can be made plausible using the Khinchin-like axioms B1–B4 introduced in this paper, but the microscopic quantum field theoretical foundations of the cosmological constant are not elucidated by this approach. The ultimate microscopic theory of a cosmological constant as embedded in a theory of quantum gravity still has to be developed.

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