# Exercise Sheet 1 — Chaos and Fractals (MTH6107)

**due: Thursday, 2 October 2008, 5pm**

1. Find all solutions of the continuous-time dynamical system

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x \end{aligned}$$

for a given initial condition $x(0) = x_0, y(0) = y_0$. Hint: Try a linear combination of sin and cos functions.

Draw the phase portrait.

2. Sketch (or plot accurately) graphical analyses of the first 4 iterations of each of the following maps $f$, with the given initial value $x_0$. For each map describe the limiting behaviour of the $n$th point of the orbit as $n \to \infty$.

a) $f(x) = x^2$, $x_0 = 0.8$

b) $f(x) = 1 - x^2$, $x_0 = 0.3$

c) $f(x) = x^x$, $x_0 = 0.25$

3. (*Numerical exercise)* Write a MAPLE program (or use any other language, e.g. FORTRAN, C++,...) that iterates the logistic map $x_{n+1} = 1 - \mu x_n^2$ about 10000 times. Choose, for example, the initial value $x_0 = 0.3$ and iterate for the following parameter values:

a) $\mu = 1.2$

b) $\mu = 1.8607825$

c) $\mu = 1.7548777$

d) $\mu = 1.5437$

e) $\mu = 1.8848036$

f) $\mu = 2$

Run the program and print out the last 20 iterates. For which of the above values of $\mu$ do you find a stable periodic orbit? In case there is a stable periodic orbit, what is the period length?