Exercise Sheet 2 — Chaos and Fractals (MTH6107)

due: Thursday, 9 October 2008, 5pm

Consider the *modified logistic map* defined by

$$x_{n+1} = f(x_n) = ax_n(1 - x_n)$$

on the phase space $X = [0, 1]$. $a \in [0, 4]$ is a parameter.

1. Sketch the function $f(x)$ for $x \in [0, 1]$ for the parameter values $a = 2, 3, 4$. Also, sketch the function $f^2(x)$ for the same parameter values. Mark the fixed points of f and f^2 in your plots. By graphical analysis, illustrate that for $a = 2$ the dynamical system has an attracting fixed point, whereas for $a = 4$ it has not. What happens at $a = 3$?

2. Derive a formula for the fixed points of f as a function of a. For each of the fixed points you find, determine the region of the parameter a where it is stable.

3. Show that for the modified logistic map a periodic orbit of length L is superstable if $x = \frac{1}{2}$ is an orbit element.

4. Determine the parameter values where the modified logistic map has a superstable periodic orbit of length 1. Evaluate the coefficients of the polynomials in a whose roots give the parameter values where the modified logistic map has superstable periodic orbits of period 2 and 3.