Exercise Sheet 3 — Chaos and Fractals (MTH6107)

due: Thursday, 16 October 2008, 5pm

1. Consider the map $f : [0, 1] \to [0, 1]$ defined by

$$f(x) = \begin{cases} x + 2x^2 & 0 \le x < \frac{1}{2} \\ 2 - 2x & \frac{1}{2} \le x \le 1 \end{cases}$$

   a) Prove that the map has no stable fixed point.

   b) Show by graphical analysis how a typical orbit evolves near $x = 0$.

   c) Show that the function $f(x)$ satisfies for x in a small vicinity of 0

$$\alpha f(f(\frac{x}{\alpha})) = f(x) \qquad (x \to 0).$$

   Show that the rescaling factor α is different from the Feigenbaum constant for this particular map.

2. Consider the function $g : \mathbf{R} \to \mathbf{R}$ given by

$$g(x) = \frac{1}{\frac{1}{x} - b}$$

   where b is some constant.

   a) Calculate $g(0) = \lim_{x \to 0} g(x)$ and $g'(0)$.

   b) Show that g is an exact solution of the Feigenbaum-Cvitanovic equation

$$\alpha g(g(\frac{x}{\alpha})) = g(x).$$

   What do you get for the rescaling factor α in this case?

3. Prove that the doubling operator R is nonlinear.