## Exercise Sheet 7 — Chaos and Fractals (MTH6107) due: Thursday, 20 November 2008, 5pm

1. For the binary shift map  $x_{n+1} = f(x_n) = 2x_n \mod 1$  on X = [0, 1], calculate time averages

$$\bar{Q} = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} Q(x_n)$$

of the following observables

- a)  $Q(x) = x^3$
- b)  $Q(x) = e^{-x}$
- c)  $Q(x) = \log |f'(x)|.$

Make use of the fact that the map is ergodic.

2. For the cusp map  $x_{n+1} = f(x_n) = 1 - 2\sqrt{|x_n|}$  on X = [-1, 1], calculate time averages of the same observables as in question 1.

Hint: Use the results of question 3, exercise sheet 6. To evaluate the integrals, use the product rule  $\int u'v = uv - \int uv'$ .

3. Prove that the Perron-Frobenius operator of an arbitrary map f is a linear operator.