## Exercise Sheet 9 — Chaos and Fractals (MTH6107) due: Thursday, 4 December 2008, 5pm

- 1. i) Determine the fractal dimension of a) the Koch curve b) the Menger sponge.
  - ii) Construct a fractal with fractal dimension  $D_0 = \log 5 / \log 3$ .
- 2. Prove that for arbitrary (normalized) probabilities  $p_i$  the Renyi dimensions satisfy  $D(q) \ge 0$ .
- 3. The Mandelbrot set M of the complex logistic map  $z_{n+1} = z_n^2 + c$ ,  $z_n \in \mathbf{C}$  is defined as

 $M = \{ c \in \mathbf{C} | z_n \text{does not go to infinity for } n \to \infty \}.$ 

Show that if c = a + ib is in the Mandelbrot set, then  $c^* = a - ib$  is also in the Mandelbrot set.

Hint: Iterate the complex logistic map with initial value  $z_0 = 0$  and parameters c and  $c^*$  and compare the iterates.