## MTH4100

Exercise sheet 1

## Calculus 1, Autumn 2009

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These questions are designed to help you understand the material covered in week $n, n \in \mathbb{N}$ lectures. Exercise sheets will typically be handed out in the Tuesday lecture of week $n+1$. You will get help on them in the exercise class on Wednesday of the same week. You should write up your solution to the starred question $(*)$ clearly and hand it in to your personal tutor during your week $n+2$ exercise class for feedback. Put your full name and student number on the top of your solution. It is important that you try to do all of the numbered questions. The extra question is for the more ambitious students.

1. Prove that

$$
\left|\frac{a}{b}\right|=\frac{|a|}{|b|}
$$

for $a, b \in \mathbb{R}, b \neq 0$.
${ }^{(*)} 2$. Determine the set of all real numbers $x \in \mathbb{R}$ that satisfy

$$
x^{2}-3 x-4<0
$$

(a) by solving the inequality, and
(b) by plotting the graph of $y=x^{2}-3 x-4$.
3. Determine the set of all real numbers $x \in \mathbb{R}$ that satisfy

$$
|2 x-1|+|4 x+1|<3
$$

(a) by solving the inequality, and
(b) by plotting the graph of $y=|2 x-1|+|4 x+1|$.
4. Determine the set of all real numbers $x \in \mathbb{R}$ that satisfy

$$
\sqrt{1-x^{2}} \leq-x
$$

(a) by solving the inequality, and
(b) by plotting the graphs of $y=-x$ and $y=\sqrt{1-x^{2}}$.

Extra: Prove the arithmetic-geometric mean inequality

$$
\sqrt{a b} \leq \frac{1}{2}(a+b) \quad, \quad a, b \geq 0
$$

