

MTH4100 Exercise sheet 1 Calculus 1, Autumn 2009 Rainer Klages

These questions are designed to help you understand the material covered in week  $n, n \in \mathbb{N}$ lectures. Exercise sheets will typically be handed out in the Tuesday lecture of week n+1. You will get help on them in the exercise class on Wednesday of the same week. You should write up your solution to the starred question (\*) clearly and hand it in to your personal tutor during your week n+2 exercise class for feedback. Put your *full name and student number* on the top of your solution. It is important that you try to do all of the numbered questions. The extra question is for the more ambitious students.

1. Prove that

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

for  $a, b \in \mathbb{R}$ ,  $b \neq 0$ .

(\*)2. Determine the set of all real numbers  $x \in \mathbb{R}$  that satisfy

$$x^2 - 3x - 4 < 0$$

(a) by solving the inequality, and

- (b) by plotting the graph of  $y = x^2 3x 4$ .
- 3. Determine the set of all real numbers  $x \in \mathbb{R}$  that satisfy

$$|2x - 1| + |4x + 1| < 3$$

- (a) by solving the inequality, and
- (b) by plotting the graph of y = |2x 1| + |4x + 1|.
- 4. Determine the set of all real numbers  $x \in \mathbb{R}$  that satisfy

$$\sqrt{1-x^2} \leq -x$$

- (a) by solving the inequality, and
- (b) by plotting the graphs of y = -x and  $y = \sqrt{1 x^2}$ .

Extra: Prove the arithmetic-geometric mean inequality

$$\sqrt{ab} \le \frac{1}{2}(a+b) \quad , \quad a,b \ge 0$$