MTH4100 Calculus 1, Autumn 2009

Exercise sheet 8

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1. **L'Hôpital or not L'Hôpital?**

   Find the following limits:

   a. $\lim\limits_{x \to 2} \dfrac{\sqrt{x^2+12}-4}{x-2}$ [2007 exam question]

   (*)b. $\lim\limits_{x \to 0} \dfrac{1-\cos(6x)}{36x^2}$ [2008 exam question]

   c. $\lim\limits_{x \to \infty} \dfrac{\sqrt{x+5}}{\sqrt{x}+5}$

2. **Estimating with finite sums.**

   Graph the function $f(x) = x^2 - 1$ over the interval $[0, 2]$. Partition the interval into four subintervals of equal length. Then add to your sketch the rectangles associated with the (Riemann) sum $\sum_{k=1}^{4} f(c_k)\Delta x_k$, given that c_k is the (a) left-hand endpoint, (b) right-hand endpoint, (c) midpoint of the kth subinterval. Make a separate sketch for each set of rectangles.

3. **Finite sums.**

   Which formula is not equivalent to the other two?

   $$\text{(a)} \sum_{j=2}^{4} \frac{(-1)^{j-1}}{j-1} \qquad \text{(b)} \sum_{k=0}^{2} \frac{(-1)^k}{k+1} \qquad \text{(c)} \sum_{l=-1}^{1} \frac{(-1)^l}{l+2}$$

(*)4. **Limit of upper sums.**

   For the function $f(x) = 1 - x^2$ over the interval $[0, 1]$, find a formula for the *upper sum* obtained by dividing the interval $[a, b]$ into n equal subintervals. Then take the limit of this sum as $n \to \infty$ to calculate the area under the curve over $[a, b]$.

Extra: Let $f(x)$, $g(x)$ be two continuously differentiable functions satisfying the relationships $f'(x) = g(x)$ and $f''(x) = -f(x)$. Let $h(x) = f^2(x) + g^2(x)$. If $h(0) = 5$, find $h(10)$.