Rainer Klages

Max Planck Institute for the Physics of Complex Systems, Dresden Queen Mary University of London, School of Mathematical Sciences

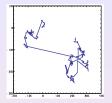
Diffusion Fundamentals VI, Technische Universität Dresden 26 August 2015



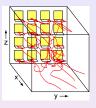




Overview







Theme of this talk:

Can search for food by biological organisms be understood by mathematical modeling?

Three parts:

- Lévy flight hypothesis: review
- Biological data: analysis and interpretation
- Foraging bumblebees: own research

Lévy flight search patterns of wandering albatrosses

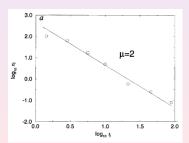
famous paper by Viswanathan et al., Nature 381, 413 (1996):

for albatrosses foraging in the South Atlantic the flight times were recorded

Overview



the histogram of flight times was fitted by a Lévy distribution (power law $\sim t^{-\mu}$)



Lévy flights in a nutshell

Lévy flights have well-defined mathematical properties:

- a Markovian stochastic process (no memory)
- with probability distribution function of flight lengths exhibiting power law tails, $\rho(\ell) \sim \ell^{-1-\alpha}$, $0 < \alpha < 2$;
- it has infinite variance, $<\ell^2>=\infty$,
- satisfies a generalized central limit theorem (Gnedenko, Kolmogorov, 1949) and
- is scale invariant

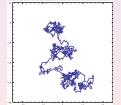
for an outline see, e.g., Shlesinger at al., Nature **363**, 31 (1993) more realistic are *Lévy walks*; Zaburdaev et al., RMP **87**, 483 (2015)

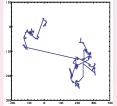
Optimizing the success of random searches

another paper by Viswanathan et al., Nature 401, 911 (1999):

- question posed about "best statistical strategy to adapt in order to search efficiently for randomly located objects"
- random walk model leads to Lévy flight hypothesis:

Lévy flights provide an *optimal search strategy* for *sparse, randomly distributed, immobile, revisitable targets in unbounded domains*





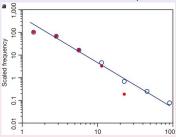
Brownian motion (left) vs. Lévy flights (right)

Lévy flights also obtained for bumblebee and deer data

Revisiting Lévy flight search patterns

Edwards et al., Nature 449, 1044 (2007):

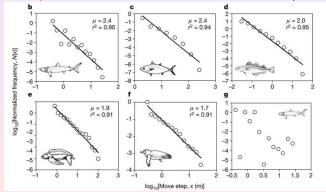
 Viswanathan et al. results revisited by correcting old data (Buchanan, Nature 453, 714, 2008):



- no Lévy flights: new, more extensive data suggests (gamma distributed) stochastic process
- but claim that truncated Lévy flights fit yet new data Humphries et al., PNAS 109, 7169 (2012) (and reply...)

Lévy **paradigm**: Look for *power law tails* in pdfs!

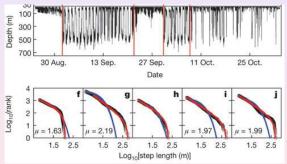
 Sims et al., Nature 451, 1098 (2008): scaling laws of marine predator search behaviour; > 10⁶ data points!



• prey distributions also display Lévy-like patterns...

Lévy flights induced by the environment?

 Humphries et al., Nature 465, 1066 (2010): environmental context explains Lévy and Brownian movement patterns of marine predators; > 10⁷ data points!; for blue shark:



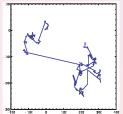
blue: exponential; red: truncated power law

 note: ∃ day-night cycle, cf. oscillations; suggests to fit with two different pdfs (not done)

Optimal searches: adaptive or emergent?

strictly speaking two different Lévy flight hypotheses:

Lévy flights represent an (evolutionary) adaptive optimal search strategy Viswanathan et al. (1999) the 'conventional' Lévy flight hypothesis



Lévy flights emerge from the interaction with a scale-free food source distribution Viswanathan et al. (1996) more recent reasoning



An alternative to Lévy flight search strategies

Bénichou et al., Rev. Mod. Phys. 83, 81 (2011):

 for non-revisitable targets intermittent search strategies minimize the search time

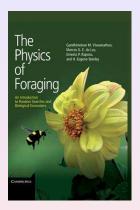


 popular account of this work in Shlesinger, Nature 443, 281 (2006): "How to hunt a submarine?"; cf. also protein binding on DNA

Summary:

Overview

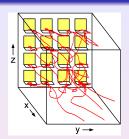
- two different Lévy flight hypothesis: adaptive and emergent
- scale-free Lévy flight paradigm
- problems with the data analysis
- intermittent search strategies as alternatives



⇒ ongoing discussions:

- mussels: de Jager et al., Science (2011)
- cells perform Lévy walks: Harris et al., Nature (2012) or not: Dieterich, RK et al., PNAS (2008)

- tracking of bumblebee flights in the lab: foraging in an artificial carpet of flowers with or without spiders
- **no test** of the Lévy hypothesis but work inspired by the *paradigm*





three experimental stages:

- spider-free foraging
- foraging under predation risk
- memory test 1 day later

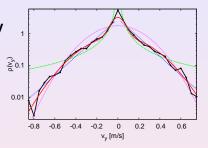
safe and dangerous flowers

Flight velocity distributions

Overview

experimental probability density (pdf) of bumblebee v_v-velocities without spiders (bold black)

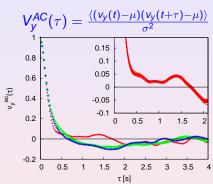
best fit: mixture of 2 Gaussians. cp. to exponential, power law, single Gaussian



biological explanation: models spatially different flight modes near the flower vs. far away, cf. intermittent dynamics

big surprise: no difference in pdf's between different stages under variation of environmental conditions!

Velocity autocorrelation function | to the wall

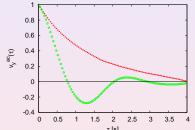


3 stages: spider-free, predation thread, memory test

all changes are in the flight correlations, not in the pdfs

model: Langevin equation

$$\frac{dv_y}{dt}(t) = -\eta v_y(t) - \frac{\partial U}{\partial y}(y(t)) + \xi(t)$$
 η : friction, ξ : Gauss. white noise



result: velocity correlations with repulsive interaction U bumblebee - spider off / on Lenz et al., PRL 108, 098103 (2012)

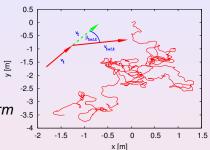
Overview

reorientation model:

Overview

describe 2d movement in comoving frame by

- speed v(t) = const.
- turning angle $\beta(t) = \xi(t)$ as random variable from *non-uniform pdf* modeling persistence



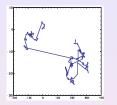
generalized model for bumblebee flights far away from flowers constructed from experimental data:

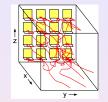
- $\beta(t) = \xi_{V}(t)$: power law correlated Gaussian noise
- $\frac{dv}{dt} = g(v(t)) + \psi(t)$: generalized Langevin equation with anti-correlated Gaussian noise

F.Lenz, A.V.Chechkin, R.K., PLoS ONE 8, e59036 (2013)

Summary

Overview

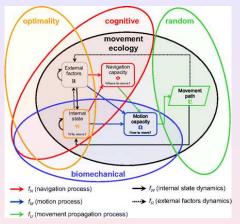






- Be careful with (power law) paradigms for data analysis.
- Other quantities may contain crucial information about foraging; example: bumblebee flights under predation thread.
- Conclusion: A more general biological embedding is needed!

Perspective: Movement Ecology Paradigm



Nathan et al., PNAS **105**, 19052 (2008)

state space approach $\mathbf{u}_{t+1} = F(\Omega, \Phi, \mathbf{r}_t, \mathbf{w}_t, \mathbf{u}_t)$ for the location \mathbf{u}_t of an organism at time t

Outlook: more to come...

Overview

Advanced Study Group on Statistical physics and anomalous dynamics of foraging MPIPKS Dresden, July - Dec. 2015



F.Bartumeus (Blanes, Spain), D.Boyer (UNAM, Mexico), A.V.Chechkin (Kharkov, Ukraine), L.Giuggioli (Bristol, UK), convenor: RK (London, UK), J.Pitchford (York, UK)

ASG webpage: http://www.mpipks-dresden.mpg.de/~asg_2015

Acknowledgement - bumblebee research: joint work with F.Lenz, T.Ings, A.V.Chechkin and L.Chittka