Weak Chaos, Anomalous Diffusion, and Fluctuation Relations

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Concluding this conference with my lectures, I do not aim at giving a basic introduction to its main themes. I assume that this has been done in previous presentations already. Rather, I will try to talk about topics that might not have been covered so far. Accordingly, please note that I may adjust this abstract based on what has been discussed at the conference in the previous weeks.

The first lecture is about **deterministic random walks** on the line generated by simple chaotic maps [1]. For the case of normal deterministic diffusion two different methods to exactly calculate the diffusion coefficient of such maps are outlined: One employs the Taylor-Green-Kubo formula for diffusion by evaluating it in terms of fractal generalized Takagi functions. The other one is based on solving eigenvalue problems of Frobenius-Perron operators and leads to the escape rate formula for diffusion, which expresses transport coefficients in terms of chaos quantities [2]. I then generalize these models towards an intermittent map exhibiting anomalous (sub)diffusion. Lacking similar methods for exactly calculating the generalized diffusion coefficient for this model, I use stochastic continuus time random walk theory and computer simulations to obtain results [1,2].

The second lecture introduces to **fluctuation relations** (FRs), which generalize the Second Law of thermodynamics to small systems. I first illustrate this concept for the ordinary Langevin equation. I then remind of three generic types of stochastic dynamics generating anomalous diffusion: Lévy flights, long-correlated Gaussian processes and time-fractional kinetics [3]. For these different models, which exhibit both sub- and superdiffusion, FRs will be checked in the simple nonequilibrium situation of a particle subject to a constant force [4]. It turns out that there is an interesting interplay between validity of fluctuation-dissipation relations and FRs. I finally show that these findings are important for understanding fluctuations in experimentally accessible systems, such as migrating biological cells and a paradigmatic lattice gas modeling glassy dynamics.

The first lecture is based on joint work with G.Knight, P.Gaspard, J.R.Dorfman, N.Korabel, and A.V.Chechkin [1,2], the second one elaborates on research performed with A.V.Chechkin [4]. Their collaboration with me on these themes is gratefully acknowledged.

[1] R.Klages, *Microscopic Chaos, Fractals and Transport in Nonequilibrium Statistical Mechanics* (World Scientific, Singapore, 2007).
[2] R.Klages, *From Deterministic Chaos to Anomalous Diffusion* (book chapter in Reviews of Nonlinear Dynamics and Complexity Vol. 3, H.G.Schuster (Ed.), Wiley-VCH, Weinheim, March 2010), p.169-227.
[3] R. Klages, G.Radons, I.M.Sokolov (Eds.), *Anomalous transport* (Wiley-VCH, Weinheim, 2008).
[4] A.V.Chechkin, R.Klages, J.Stat.Mech. L03002 (2009)