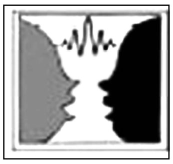
Face to Face



This section features conversations with personalities related to science, highlighting the factors and circumstances that guided them in making the career choice to be a scientist.

In Conversation with a Global Mathematician

Peter J Cameron talks to Amrita Antony

Peter J Cameron, born on 23rd January 1947 in Toowoomba, Australia, graduated from the University of Queensland and obtained a DPhil from Oxford University, UK, where he was a Rhodes Scholar. Currently he is a Professor of Mathematics and Director of Pure Mathematics, in the School of Mathematical Sciences at Queen Mary, University of London. He was awarded the London Mathematical Society's Junior Whitehead Prize in 1979, and was a speaker at the International Congress of Mathematicians in 1990. He also spent a term at the California Institute of Technology, USA as Sherman Fairchild Distinguished Scholar.

Prof. P J Cameron is an interesting and multifaceted personality, and it was indeed a privilege to interview him. Here is the extract of the interview.

AA: You have come all the way from Queen Mary, University of London to Cochin, to give a plenary talk in the 'International Conference on Recent Trends in Graph Theory and Combinatorics'– a Satellite Conference of ICM-2010. What is the difference you find abroad and here at Cochin, especially in terms of audience response?

PJC: Really not a lot of difference – Mathematicians are the same anywhere. I feel that I have to put on a good show, to get them interested – it is a bit like being an actor, and they will listen to what I have to say and react to it. It was a very good audience, because the conference ran so smoothly and everyone was happy.

AA: What inspired you to take up this branch of science? Is there any childhood experience that led you to mathematics?

PJC: I was always interested in mathematics, from early childhood. I grew up on a dairy farm, and I have two early memories: We used to take the milk to the cheese factory in a horse-drawn cart; I sat on the back of the cart, counting up to 1000. One day at primary school while I was

Amritha Antony interviewing.

chasing the cows in to be milked, I suddenly saw how to find the sum of a geometric progression with common ratio 3.

AA: You have said "Like the hero of Richard Brautigon's novel (*Box* 1), I like to count things!" What led you to count things, especially infinite structures?

PJC: I started thinking about infinite structures in the 1970's. I had been a finite group theorist before that time, and by then it was clear that the finite simple groups would soon be classified, so I was looking for something else to do. But of course, I was a bit scared of the infinite; I found an area where I could study the infinite by looking at the finite parts of it and counting how many different things can occur. This gives interesting sequences of integers, whose growth rates tell us something about the infinite structure.

AA: Do you still have a picture of any particular tutor or faculty from your primary classes who had led you to this present stage?

PJC: One teacher at primary school influenced me, in a way he probably didn't mean to. He didn't like me very much. One day, when I was sick, he taught the class square roots, and when I returned he set a test on them. I had to figure out for myself how to do square roots! I got a lot of confidence from doing this.

AA: What about students? Do you hold anyone close to your heart, someone who shares your own views?

PJC: I have had some outstanding students; I think I have learned more from my students than they have from me! It is hard to single one out, but perhaps one close to my heart was the Russian mathematician Dima Fon-Der-Flaass. (He was not actually my student but he worked

Box 1. Richard Barautigan's Novel

Richard Brautigan was a 20th century American novelist, poet, and short story writer. His work often employs black comedy, parody, and satire. He is best known for his 1967 novel *Trout Fishing in America*. Brautigan is noted for his whimsical novels that, by his own account, evolved from his efforts to write poetry. The hero of his novel *The Hawkline Monster* is a gunslinger called Cameron whose trademark is that he counts everything.

with me as a post-doc.) Before I met him, a mutual friend described him to me as "your mathematical son", since like me, Dima was interested in everything, and could think about any problem he was given. Dima died earlier this year, which was a tragedy for his friends.

AA: May I have the details about the Indian students working under you?

PJC: I have never had an Indian postgraduate student, though I have had students from many countries including Brazil, New Zealand and USA.

AA: You have spoken at ICM (*Box* 2) 1990; do you feel that you will lose something great by missing the ICM at Hyderabad?

PJC: Probably I did, yes. But I am the sort of person who prefers small conferences, where you can get to know people and really work with them. My memory of Kyoto in 1990 is that it was very difficult even to find the person you wanted to speak to among the thousands of mathematicians there. Anyway it was a choice between having a holiday in Kerala, "God's Own Country", or going to the ICM.

AA: In California Institute of Technology you have spent a term as 'Sherman Fairchild Distinguished Scholar'. Can you explain what it is all about?

PJC: This was arranged by the famous group theorist Michael Aschbacher, whom I knew from his visit to Oxford. I was able to spend a semester at Caltech, living in a huge old house, with no specific obligations. But I cannot just sit and do nothing, so I volunteered to give a course of lectures anyway. There were also some very interesting people to work with – like Michael Aschbacher, and I worked with combinatorialists such as Richard Wilson, Peter Frankl and Phil Hanlon.

AA: A quick explanation about oligomorphic permutation groups?

PJC: Suppose G is a group of permutations of an infinite set and suppose that the n-tuples from an infinite set fall into only finitely many orbits under the action of G. Then G is said to be

Box 2. International Congress of Mathematicians

The International Congress of Mathematicians (ICM) is the most prestigious and the largest congress in the mathematics community. It is held once every four years under the auspices of the International Mathematical Union (IMU). The Fields Medals, the Nevanlinna Prize, and the Gauss Prize are awarded during the congress' opening ceremony. In the ICM–2010 held in Hyderabad, India, a new international award, S S Chern Medal, was also awarded..

oligomorphic. Consider the group of all order-preserving permutations of the rational numbers. You can map any n-tuple to any other provided they are in the same order. So the number of orbits is $n!$, the number of different orderings of n points. There is a very important theorem in logic that a countable structure is categorical (that is, it is specified by first-order axioms) if and only if its automorphism group is oligomorphic. So, for example, rational numbers are countable and are categorical. This is Cantor's theorem saying that the rational numbers form the unique countable dense ordered set without endpoints. Such groups have traditionally been linked with model theory and combinatorial enumeration; more recently their group-theoretic properties have been studied, and links with graded algebras, Ramsey theory, topological dynamics, and other areas have emerged.

AA: About your future plans?

PJC: I have a lot of projects at the moment: synchronization (which I talked a bit about at the Cochin conference), optimal design theory in statistics, algebraic properties of chromatic roots, etc.

AA: What is your opinion about Kerala, its cultural heritage? And about Kerala's mathematical history (*Box* 3)?

Box 3. Mathematical Heritage of Kerala

The Kerala school of astronomy and mathematics was founded by Madhava of Trichur District. Kerala The school included among its members: Parameshvara, Neelakanta Somayaji, Jyeshtadeva, Achyuta Pisharati, Melpathur Narayana Bhattathiri and Achyuta Panikkar. It flourished between the 14th and 16th centuries and the original discoveries of the school seem to have ended with Narayana Bhattathiri (1559–1632). In attempting to solve astronomical problems, the Kerala school independently created a number of important mathematics concepts. Their most important results – series expansion for trigonometric functions – were described in Sanskrit verse in a book by Neelakanta called *Tantrasangraha*, and again in a commentary on this work, called *Tantrasangraha-vakhya*, of unknown authorship. The theorems were stated without proof, but proofs for the series for sine, cosine, and inverse tangent were provided a century later in the work *Yuktibhasa* (1500–1610), and also in a commentary on *Tantrasangraha*.

The Indian system of learning was totally different from that of Europe during the time of Madhava. However, things changed in the 16th century, when Jyestadevan wrote his book *Yuktibhasa* in his mother tongue Malayalam, with statement of proof of the findings of his Guru Parampara (earlier scholars) including that of Madhava. Their work, completed two centuries before the invention of calculus in Europe, provided what is now considered the first example of a power series (apart from geometric series). However, they did not formulate a systematic theory of differentiation and integration, nor is there any direct evidence of their results being transmitted outside Kerala.

PJC: I was very interested with everything I read or saw about Kerala, which seems quite different from other parts of India I had visited before. It has such a high literacy rate and good public health and very great achievements: there is a great deal of history as well, especially mathematical history. I had read about the 16th centuary Kerala School of mathematics earlier, and I learned a lot more during this visit, especially about Madhavan, whose amazing work on infinite series for trigonometric functions was without equal anywhere in the world at that time. It was also interesting to see, in Fort Cochin, evidence of the many different civilizations (Chinese, Arab and European) who had left their marks on Kerala.

AA: What about your family? Who all are there in your family and what are they doing?

PJC: I have three children, all married now. (My daughter got married only this month, so I was able to get a wedding present for her from Kerala) and three grand children. My daughter and her husband live in London, and my sons with their families live in towns not too far away from mine, so I am able to see them quite often. None of my children is a mathematician; may be this was their way of rebelling against their father!! My daughter works for an oil company, one son is a management accountant and the other makes a living drawing comics.

AA: What about your hobbies? Your favourite poet and music band?

PJC: My hobby now is walking; every weekend I try to spend at least one day on a long walk. I wish that my schedule in Kerala had allowed me to do a bit more walking! Also I love music; I play guitar. One of my favourites (I count him as both a poet and a musician) is Bob Dylan. Another favourite poet is T S Eliot. I am interested in almost all kinds of music; I came to love Indian music when the Beatles dabbled in it in the 1960's and one of the highlights of my visit to Culcutta in 1988 was meeting the violinist V G Jog, one of whose record I bought when I was a student in 1965.

AA: As the chair of British Combinatorial Committee, could you explain how it benefits the mathematicians all over the world?

PJC: The BCC is a registered charity whose purpose is to support combinatorics in Britain. But mathematics is international, and I think that what we do is helpful to everybody. We run a big international conference every two years and more than half of the delegates are from other countries. Also we keep a list of conferences in combinatorics and related areas on the web; this is used by mathematicians everywhere. We also work behind the scenes to persuade other mathematicians and funding bodies of the importance of combinatorics!

AA: Do you think the above mentioned benefits reach all the mathematicians in every corner of the world. If not, how could you attain it?

PJC: The web is the best way to reach people now, and that is where we put our resources. But there is no substitute for actually travelling and meeting people in other countries. If we had more money we would probably use it to support international travel and collaborations.

AA: What are the recent trends in combinatorics (*Box* 4)?

PJC: One thing that has been happening for some time now is that combinatorics is more part of the mathematical community; other mathematicians have always used combinatorial arguments in their work but now they are happy to say so. The subject links with algebra, probability, computer science, dynamics, indeed almost every area of mathematical sciences. Within combinatorics, one development is that problems about graph theory are being exteneded to

Box 4. Combinatorics and Graph Theory

Combinatorics is a branch of mathematics concerning the study of finite or countable discrete structures. Aspects of combinatorics include counting the structures of a given kind and size deciding when certain criteria can be met. Combinatorial problems arise in many areas of pure mathematics, notably in algebra, probability theory, topology, and geometry. Many combinatorial questions have historically been considered in isolation, giving an *ad hoc* solution to a problem arising in some mathematical context. India is a country with a great scientific and mathematical heritage since the Vedic times (5000 BC) and which still continues to be in the forefront of many disciplines. With the decline of the Roman empire the center of mathematical research began to shift to India. India had a rich tradition of 'combinatorial thoughts' also since the works of the ancient scholar Pingala (3rd century BC). In his *Chandas- Sutra* he considers the method of finding the number of combinations obtainable by taking one, two, etc. letters out of a given number of letters. He mentions a term, 'Meru Prastara' which is the same as the modern Pascal's triangle. Mahavira (650 AD) also had some contributions to the theory of combinations in his magnum opus *Ganithasara Samgraha*.

N L Biggs, E K Lloyd and R J Wilson have remarked: "It is strange that there is almost no material relevant to combinatorics in the literature of the classical Western civilization. All the evidence points to the fact that the originators of the subject came from the East. The main stimulus came from the Hindus". It is also known that the modern concepts of permutations and combinations – formulas for the number of permutations of an n-set and the number of k-subsets of an n-set, date back to Bhaskara (1114–1185 AD) and Brahmagupta (6th century) respectively.

In the latter part of the twentieth century however, powerful and general theoretical methods were developed, making combinatorics into an independent branch of mathematics in its own right. One of the oldest and most accessible parts of combinatorics is graph theory. Graph theory as a mathematical discipline was initiated by Leonhard Euler in his discussion on the Konigsberg Bridge Problem. Today, graph theory is a branch of mathematics which finds application in many areas. More recently, web graphs have been finding applications in very many different areas.

hypergraphs. They tend to be much harder, but a lot of good work is going on in tackling questions about extremal hypergraphs, colourings, etc.

AA: As your current interest is on the connection between optimal design and Laplace eigenvalues of multigraphs, could you explain what draws your interest to it?

PJC: I began my research career working on combinatorial design theory; I knew that the subject had its roots in statistics (with the work of Fisher, Bose, and others) but until fairly recently I didn't know what the connections were. It turns out that the designs that mathematicians studied are not general enough for the statisticians, who have to do the best they can even if the parameters don't satisfy some restrictive conditions. The other thing is that it is very beautiful mathematics! Who would have suspected that the notion of A-optimality for block designs was connected to electrical networks, random walks, and other important topics in graph theory?

AA: What are the recent researches in design theory?

PJC: One of the threads is what I have just talked about. But one big development is the theory of trades in designs, which tell you how to change some blocks to turn one design into another. These have opened the possibility of choosing a random design (in cases where there are many to choose from, e.g. Steiner triple systems (*Box* 5)). This is something that statisticians have been interested in since the days of Fisher and Yates.

AA: Could you give us a brief explanation about 'Mathematical Genealogy Project'.

PJC: This project began when a few enthusiasts decided to collect information about mathematicians and their "academic fathers". This usually meant their PhD advisers. However, when you go back into the past, there were no formal PhDs, and so it is not so clear who somebody's academic father is. But they have recently been putting a lot of historical data about mediaeval and renaissance mathematics in Europe into the database. I hope that this will extend to other parts of the world; in those days, mathematics was not universal in the way it is now. We talked about the Kerala mathematics school; I believe that this data should be included. George Ghevarghese Joseph suggested in his book about non-European mathematics that perhaps the tradition of mathematics in south India could have been kept alive long enough to have influenced Ramanujan, even though there is no documentation of this.

AA: Your opinion about current mathematical developments and applications in our life?

Box 5. Steiner Triple Systems

Steiner systems were introduced by Kirkman in 1847 by Thomas Kirkman. Triple systems were introduced and further studied independently in 1853 by J Steiner, whence the name.

Let X be a set of $v \geq 3$ elements together with a set B of 3-subset (triples) of X such that every 2-subset of X occurs in exactly one triple of B. Then B is called a Steiner triple system and is a special case of a Steiner system with $t = 2$ and $k = 3$. A Steiner triple system $S(v) = S(v, k = 3, \lambda = 1)$ of order v exists iff $v \equiv 1,3 \pmod 6$ (Kirkman 1847). In addition, if Steiner triple systems S_1 and S_2 of orders v_1 and v_2 exist, then so does a Steiner triple system S of order $v_1 v_2$.

Examples of Steiner triple systems $S(v)$ of small orders v are

$$S_3 = \{\{1, 2, 3\}\}$$
$$S_1 = \{\{1, 2, 4\},\{2, 3, 5\},\{3, 4, 6\},\{4, 5, 7\},\{5, 6, 1\},\{6, 7, 2\},\{7, 1, 3\}\}$$
$$S_9 = \{\{1, 2, 3\},\{4, 5, 6\},\{7, 8, 9\},\{1, 4, 7\},\{2,5,8\},\{3,6,9\},\{1,5,9\},\{2,6,7\},\{3,4,8\},\{1,6,8\}, \{2,4,9\}, \{3,5,7\}\}$$

PJC: So much of modern life rests on a basis of mathematics: number theory keeps our communications safe; linear algebra and combinatorics keep them free from errors; graph theory organises our commercial life, while stochastic differential equations decide on the price of financial assets. In Britain, young people are increasingly aware that mathematics is a good subject to study career-wise. I think that the future is bright for our subject!

That 20-minute conversation ended on a warm note wherein we understood that this great mathematician is a soft spoken, down to earth man who loves not only his subject but also mankind as a whole – “A true global mathematician!” We cherish the moments shared with him with great reverence and on his part he described his memories about Kerala as ‘golden’. As he said, we shall meet again if God wills!

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